USER GUIDE

C2D2E : Non-LTE 2D radiative transfer in cylindrical threads with incident radiation (hydrogen + helium)

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1 Introduction

The C2D2E code was written in Fortran 77 by P. Gouttebroze ([16]) and describes Non-LTE (NLTE) radiative transfer (2D) for hydrogen and helium atoms in a cylindrical thread inside the solar corona, with incident radiation, using complete frequency redistribution. We assume that the cylinder is filled with a mixture of hydrogen and helium. We treat NLTE radiative transfer and statistical equilibrium of level populations for both atoms in 2D. These equations (of radiative transfer and statistical equilibrium) are solved using cylindrical coordinates. Electron density is obtained by ionization equilibrium of these 2 atoms.

These hydrogen and helium lines (intensity, profiles) are used to diagnose solar prominences through comparison with observations.

The atmosphere model considered here is the same as in the CYMA2DV code (2D NLTE radiative transfer for hydrogen atom in a cylinder with incident radiation and 3D velocity fields, [13]). For more details, see the documentation of CYMA2DV code.

The original version of C2D2E, adapted to **gfortran** compiler is available from MEDOC website:

https://idoc.ias.u-psud.fr/MEDOC/Radiative transfer codes

In the following sections, we will explain in details C2D2E (hydrogen + helium): modeling, implemented equations, algorithm, numerical methods, etc.

2 Description of C2D2E code

Nature of the physical problem: NLTE radiative transfer (2D) in a cylinder with incident radiation (hydrogen and helium lines)

Method of solution: Λ -iteration method + preconditioning (MALI)

Other relevant information: * Two types of models:

- model whose parameters don't vary inside the cylinder, defined by DIAM, TMOD, PG-MOD, ALDEG, HAL (cylinder diameter (km), temperature (°K), gas pressure (dyn.cm⁻²), angle of inclination of the cylinder (in degree), altitude (km))
- model whose parameters vary inside the cylinder, defined by PGMOD, ALDEG, HAL, R0KM, R1KM, T0, T1 (gas pressure (dyn.cm⁻²), angle of inclination of the cylinder (in degree), altitude (km), internal radius (km), external radius (km), temperature (°K) inside R0KM, temperature (°K) between R0KM and R1KM).
- * Complete frequency redistribution (CRD) is used for hydrogen and helium lines and continua

Author: P. Gouttebroze

Program available from:

https://idoc.ias.u-psud.fr/MEDOC/Radiative transfer codes

Computer(s) on which program has been tested: PC with 4 Intel processors (2.67GHz)

Operating System(s) for which version of program has been tested: Linux

Programming language used: Adapted in Fortran 90 for gfortran compiler

Status: Stable

Accessibility: open (MEDOC)

No. of code lines in combined program and test deck: c2d2e.f90 (\sim 4000 lines), visu.f90 (\sim 2000 lines)

Typical running time: 1h30 for a given model whose parameters vary in the cylinder

References:

- P. Gouttebroze, "Radiative transfer in cylindrical threads with incident radiation. VI : A hydrogen plus helium system", A&A 503, 663-671, 2009
- P. Gouttebroze, "Radiative transfer in cylindrical threads with incident radiation. III : Hydrogen spectrum", A&A, 448, 367-374, 2006

3 Algorithm

Figure 1 describes the algorithm of C2D2E code.

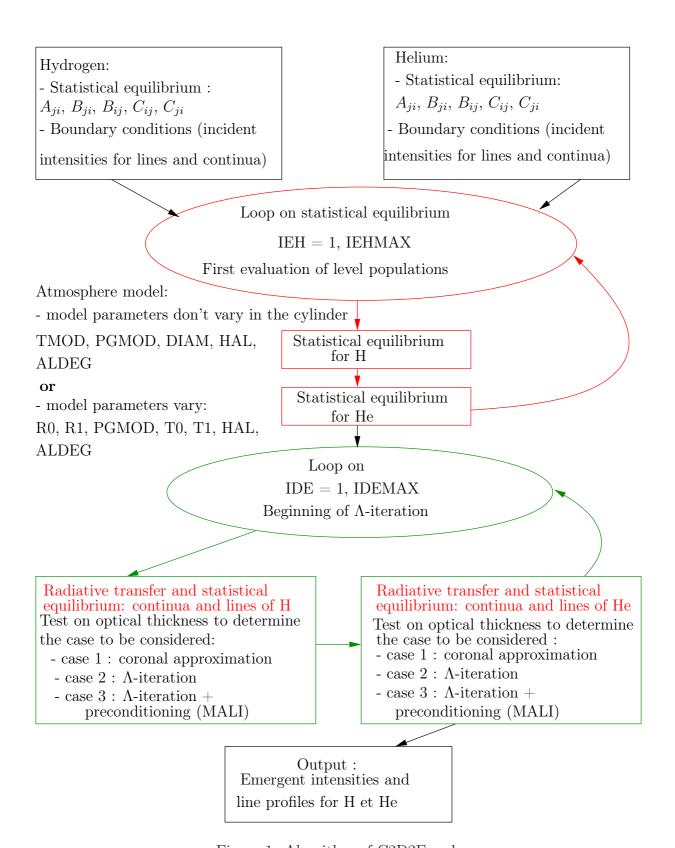


Figure 1: Algorithm of C2D2E code

4 Cylindrical thread modeling

The modeling in C2D2E code is the same as in CYMA2DV code ([13]) except that there is no 3D velocity fields but helium atom and its ionization states are treated here. For more details on cylindrical thread modeling, see the documentation of CYMA2DV.

5 Bound-Free and Bound-Bound transitions

The transitions used in C2D2E code are Bound-Free and Bound-Bound transitions, which are represented in figure 2. Bound-Free transitions are between a bound state i and a continuum, producing a free electron with energy ϵ . It starts from excited states limit, i.e. $\epsilon = 0$. Bound-Bound transitions are from level i to level j.

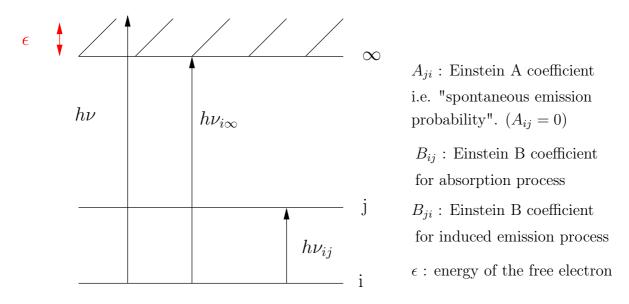


Figure 2: Transition types used in C2D2E

6 Atomic data

In this section we describe the atomic structure of the following atoms: hydrogen and helium.

6.1 Hydrogen (Johnson model, [20])

The transitions for hydrogen are summarized in Table 1. NN is the number of energy levels, including the treshold level continuum, bound levels limit (∞) . $NTT = NN \times (NN-1)/2$ is the total number of transitions, NTRC = NN - 1 is the number of continua (bound-free transitions) and NTAR = NTT - NTAC is the number of lines (bound-bound transitions). In C2D2E, NN = 6 (5 levels and 1 continuum).

Transition	Lower level	Upper level	Transition names	Wavelength $\lambda(A)$
1	1	2	$Ly - \alpha$	1215
2	1	3	$Ly - \beta$	1025
3	2	3	$H-\alpha$	6564
4	1	4	$Ly - \gamma$ $H - \beta$	972
5	2	4		4862
6	3	4	$Pa - \alpha$	18756
7	1	5	$Ly - \delta$	949
8	2	5	$H-\gamma$	4341
9	3	5	$Pa - \beta$	12821
10	4	5	$Br - \alpha$	6563
11	1	6	$Ly - \epsilon$	973
12	2	6	$H - \delta$	4102
13	3	6	$Pa - \gamma$	10935
14	4	6	$Br - \beta$	4861
15	5	6	$Pf - \alpha$	7460
16	1	7	Ly-6	931
17	2	7	$H - \epsilon$	3971
NTAR + 1	1	NN	Lyman Continu	911
NTAR + 2	2	NN	Balmer continu	3645
NTT =	NN-1	NN		

Table 1: Main transitions for hydrogen

6.2 Helium (Benjamin model, [5])

We consider 34 levels for helium: 29 levels for He I, 4 levels for He II and 1 level for He III ([22]). We use here Benjamin model ([5]).

7 Population equations and radiative transfer equation

Let's consider level j in figure 2. At equilibrium, the population equations for this level contain all processes of population and depopulation of level j to other levels i which are balanced:

$$N_j \sum_{j \neq i}^n P_{ji} = \sum_{j \neq i}^n N_i P_{ij}$$
 (7.1)

The depopulation rate P_{ji} from level j is written as:

$$P_{ji} = A_{ji} + B_{ji}\bar{J}_{ij} + C_{ji} = R_{ji} + C_{ji}$$
(7.2)

where C_{ji} is the collisional deexcitation rate, proportional to the electron density. R_{ji} is the radiative depopulation probability either by absorption and by spontaneous (A_{ji}) or by stimulated emission. A_{ji} and B_{ji} are corresponding to Einstein coefficients.

The population rate P_{ij} of level j can also be written as:

$$P_{ij} = B_{ij}\bar{J}_{ij} + C_{ij} = R_{ij} + C_{ij},$$

where R_{ij} and C_{ij} are similar to R_{ji} and C_{ji} , except for spontaneous emission $(A_{ij} = 0)$. The left-hand side term in the population equations (7.1) represents all the processes which depopulate level j and the right-hand side term corresponds to all the levels which populate the level j. And we use the following closure conservation equation:

$$\sum_{j=1}^{n} N_j = N_T \tag{7.3}$$

where N_T is the total population of the atomic element.

These population equations (7.1) which contain the mean intensities via \bar{J}_{ij} (7.9) are coupled with the radiative transfer equation (7.4).

The specific intensity I in the radiative transition is obtained by solving the transfer equation:

$$\frac{dI(\Delta\nu, \mathbf{n})}{ds} = k_{ij}(\Delta\nu, \mathbf{n})[S_{ij} - I(\Delta\nu, \mathbf{n})], \tag{7.4}$$

s is the abscissa along the ray, **n** is the direction of the ray, $\Delta \nu$ is the frequency difference with the frequency line center ν_{ji} , S_{ij} is the source function (independent of frequency and direction, under the assumption of complete redistribution):

$$S_{ij} = \frac{N_j \ A_{ji}}{N_i \ B_{ij} - N_j \ B_{ji}} \tag{7.5}$$

 $k_{ij}(\Delta\nu, \mathbf{n})$ is the absorption coefficient in a spectral line:

$$k_{ij}(\Delta \nu, \mathbf{n}) = k_{ij}^{M} \ \phi(\Delta \nu, \mathbf{n}), \tag{7.6}$$

with

$$k_{ij}^{M} = \frac{h \nu_{ji}}{4\pi} (N_i B_{ij} - N_j B_{ji})$$
 (7.7)

 ϕ is the normalized absorption profile (which is also the emission profile under the assumption of complete redistribution):

$$\phi(\Delta\nu, \mathbf{n}) = \frac{1}{\sqrt{\pi}\Delta\nu_D} H \left[a, \frac{1}{\Delta\nu_D} \Delta\nu \right], \tag{7.8}$$

where H is the Voigt function, c is the speed of light, $\Delta \nu_D$ the Doppler width, a the damping factor (natural broadening + collisional broadening + ...).

The mean intensity \bar{J}_{ij} is obtained by integrating the specific intensity, multipled by the absorption profile, with respect to frequency and direction:

$$\bar{J}_{ij} = \frac{1}{4\pi} \int_{-\infty}^{+\infty} d(\Delta \nu) \oint I(\Delta \nu, \mathbf{n}) \ \phi(\Delta \nu, \mathbf{n}) \ d\Omega, \tag{7.9}$$

where Ω is the solid angle.

Thus, we obtain a coupled equation system which is solved by numerical methods such as accelerated Lambda-Iteration method (MALI, [26], [24]). For more details, see the next two sections.

8 Formalism used for radiative transfer

The main phases to treat radiative transfer are:

- Initialization: determination of geometrical, physical and atomic parameters. The incident intensities are also computed for each position at the surface of the cylinder and each direction.
- First evaluation of level populations, in the optically thin approximation. By averaging the incident intensities, we obtain mean intensities in the different transitions of hydrogen and helium. From these intensities and physical parameters, we compute the radiative and collisional transition rates. Then, we solve statistical equilibrium equations to obtain atomic level populations at each point of the (r, Ψ) mesh. Since the transition rates depend on electron density, it is necessary to iterate. We start from an arbitrary value of $\omega = N_e/N_H$ (e.g. $\omega = 1/2$, where N_e and N_H are electron and hydrogen densities).

After computation of transition rates and solution of statistical equilibrium, we compute N_{HII} , N_{HeII} and N_{HeIII} by adding the populations of individual levels together, and deduce a new value of ω . These operations are repeated until convergence.

• Full iterations with radiative transfer: this is the main part. The external scheme is similar to that of the preceding step, with a variable ω controlling convergence of iterations but, in the meantime, the internal intensities for all transitions of hydrogen and helium are recomputed according to the principles of NLTE radiative transfer: absorption coefficients are derived from atomic level populations (determined in the preceding iteration). Then, intensities are computed by solving the transfer equation along each ray and integrating with respect to directions and frequency. At the same time, the diagonal terms of the Λ operator are calculated. The new intensities and the diagonal Λ coefficients are used to form preconditioned statistical equilibrium equations. These equations are solved to obtain new level populations.

9 Subroutine descriptions

C2D2E code starts with the reading of the atmosphere parameters ("paramod.dat") and the mesh parameters.

Input files for C2D2E:

- hydrog.com: definition of common variables for hydrogen
- helium.com: definition of common variables for helium
- param.com: definition of common parameters
- intinc.dat: incident intensities for hydrogen
- intinc He.dat: incident intensities for helium
- paramod.dat: model to be considered, defined by the following parameters: DIAM, TMOD, PGMOD, ALDEG, HAL, R0KM, R1KM, T0, T1, TREF (cylinder diameter (km), temperature (°K), gas pressure (dyn.cm⁻²), inclination angle of the cylinder (in degree), altitude (km), internal radius (km), external radius (km), temperature inside the internal radius (°K), temperature between R0KM and R1KM (°K), reference temperature (°K)).

Two types of models:

- * If ITVAR=0: model whose parameters don't vary inside the cylinder, defined by ALDEG, HAL, DIAM, TMOD, PGMOD
- ★ If ITVAR=1, model whose parameters vary inside the cylinder, defined by ALDEG, HAL, R0KM, R1KM, T0, T1, PGMOD, TREF. The other unused variables are set to zero.

Output files for C2D2E:

- resu2d.dat: file containing emergent intensities. It is the input data of the visualization program "visu.f90". Output: cosbou.ps (hydrogen and helium line profiles)
- fort.20: electron density
- fort.23: hydrogen level populations
- fort.25: helium level populations

In the next sections, hydrogen and helium modelings will be explained in details, such as all subroutines called by C2D2E code and a set of variables used in the code.

9.1 Set of variables used in C2D2E code

- ITVAR = 0: model whose parameters don't vary inside the cylinder, ITVAR = 1: model whose parameters vary inside cylinder
- NPSI: number of points for the angular variable PSI (Ψ)
- RR (array of size NR): radius of successive layers from the center
- TAU: optical thickness (between two points)
- NZETA: number of polar coordinates ζ
- NTHET: number of polar coordinates θ
- NFR: number of frequencies in a line
- NTAB: size of the temperature array TAB (for the computation of collision rate coefficients)
- NINF: transition lower level
- NSUP: transition upper level
- AJI: Einstein A coefficient i.e. spontaneous emission
- BJI, BIJ: Einstein B coefficients for absorption and induced emission processes
- CIJ, CJI: collisional excitation/deexcitation coefficients
- CPOL: polynomial coefficients of degree 5 used in the computations of dilution factors (array size : NCMX)
- RIJ, RJI: radiative transition rates
- FRR: frequency in the line
- Atmospheric parameters (array of size $NPSI \times NR$): TE (temperature), PG (gas pressure), VT (microturbulent velocity), HNH (hydrogen density: number of atoms per unit volume), HNE (electron density)
- HN: hydrogen level populations
- FRN: level frequency
- XFR and XFRC: division models in frequency for lines and continua
- PHIL: absorption profile
- SF: source function

- CAFR: absorption coefficient
- ITRC and ITRD: control index for lines and continua
- OIS: collisional ionization rate coefficient

9.2 Hydrogen (Johnson model, [20]) and Helium (Benjamin model, [5]) modelings

The C2D2E code is divided in several parts:

- Reading atmosphere parameters (input file: "paramod.dat")
- Definition of the geometry (mesh) of the cylinder (grid in r, grid in Ψ)
- Initialization of atmospheric parameters: temperature (TE), pressure (PG), microturbulent velocity (VT)
- MATIPSP: computation of interpolation matrix in Ψ (PSI in the subroutine)
- Definition of grids in ζ (ZETA in the subroutine) and in θ (THETA in the subroutine)
- DPAHYD: definition of atomic parameters for hydrogen
- DPAHEL: definition of atomic parameters for helium
- $do\ IEH = 1, IEHMAX$: loop on statistical equilibrium in order to determine electron density (by assuming that the medium is optically thin)
 - * OPMIHYD: solution of statistical equilibrium for hydrogen
 - * OPMIHEL: solution of statistical equilibrium for helium
 - * POPHYD: prints hydrogen level populations
 - * POPHEL: prints helium level populations
 - \star Convergence test
- \bullet enddo (IEH)
- do IDE = 1, IDEMAX: loop on Λ -iteration (Radiative transfer)
 - \star do ICR = 1, ICRMAX: internal loop
 - C2DHYD: Computation of collisional transition rates and solution of radiative transfer equations for hydrogen (lines + continua)
 - > PRFHEL: absorption coefficient profiles are computed again, by taking into account collisional broadening

- ▷ C2DHEL: equivalent to C2DHYD subroutine for helium
- > POPHYD: prints hydrogen level populations
- > POPHEL: prints helium level populations
- \star enddo (ICR)
- \star Convergence test
- \bullet enddo (IDE)
- RESUHYD: prints in output file "resu2d.dat" the values of r, Ψ , k (absorption coefficient), S (hydrogen source function)
- RESUHEL: prints in output file "resu2d.dat" emission and absorption coefficients for the main helium lines

Subroutines called before are explained in the next paragraph.

MATIPSP

Computation of interpolation matrices in Ψ (coordinate) since the points are not necessarily on a node of the grid or inside the grid.

DPAHYD

Definition of atomic parameters for hydrogen and quantities indenpendent of particle densities:

- computation of collisional excitation and ionization coefficients OIS, spontaneous emission probabilities AJI, BJI, BIJ: we use the model of L.C. Johnson ([20]). For more details, see CYMA2DV code and its documentation.
- definition of hydrogen energy levels
- frequency division for lines and continua
- definition of natural broadening constants
- computation of incident intensities for lines and continua
- computation of mean intensities for lines and coefficient RIJ in the optically thin case.

DPAHEL

Definition of atomic parameters for helium. We use Benjamin model ([5]).

OPMIHYD

Solution of statistical equilibrium for hydrogen. We first calculate collisional transition rates and radiative transition rates in bound-free transitions. Then, we solve a linear system in order to obtain hydrogen level populations.

Inputs: TE, HNE, HNH

Output: HNHII

OPMIHEL

Equivalent of OPMIHYD subroutine for helium.

Inputs: TE, HNE, HETOT

Outputs: HEII, HEIII

C2DHYD

Solution of radiative transfer for hydrogen lines and continua. 3 cases:

• case 1: coronal approximation

• case 2: simple Λ -iteration

• case 3: Λ-iteration + preconditioning (MALI)

C2DHYD uses STYC2D subroutine which computes intensity along the ray crossing the cylinder, knowing absorption coefficients (7.6), source functions (7.5) and intensities at each extremity of the cylinder. C2DHYD also uses DZTC2DV subroutine which computes the contribution to the diagonal of the Λ operator for the cylinder from Fourier components of the absorption coefficient along a trajectory.

PRFHEL

Computation of absorption coefficient profiles taking into account collisional broadening.

C2DHEL

Equivalent to C2DHYD subroutine for helium.

10 Results for hydrogen and helium lines

We consider a given model whose parameters vary inside the cylinder:

 $PGMOD = 0.1 \text{ dyn cm}^{-2}, ALDEG = 0^{\circ}, HAL = 0 \text{ km}, R0KM = 500 \text{ km}, R1KM = 1000 \text{ km}, T0 = 6000 \text{ K}, T1 = 10^5 \text{ K}, TREF = 10^4 \text{ K}.$

The following figures for hydrogen and helium lines are stored in the folder **results** when you download the package source file C2D2E.tgz from MEDOC website.

Figure 3 represents relative intensities (top) and absolute intensities (bottom, in erg cm⁻² s⁻¹ sr⁻¹ Å⁻¹) for the following hydrogen lines: $L\alpha$ (green), $L\beta$ (blue), $H\alpha$ (red), $L\gamma$ (black), $H\beta$ (yellow), $P\alpha$ (orange), with respect to the position along an axis perpendicular to the cylinder.

Figure 6 represents relative intensities (top) and absolute intensities (bottom) for the following helium lines: $He~I~(10830\text{\AA}, \text{green}), He~I~(584\text{Å}, \text{blue}), He~I~D3~(\text{red}), He~I~(6878\text{Å}, \text{black}), He~I~(537\text{Å}, \text{yellow}), He~II~(304\text{Å}, \text{orange}), with respect to the position along an axis perpendicular to the cylinder.$

Figures 4 (resp. figure 7) represent intensity variations accross the bottom of the loop for 10 equidistant positions (every 200 km) for hydrogen (resp. helium). The intensity scale for each graph is indicated at left, in erg cm⁻² s⁻¹ sr⁻¹ Å⁻¹. Relative wavelengths (Å) are represented in abscissae ($\Delta\lambda$).

Figures 5 and 8 represent line profiles for hydrogen and helium.

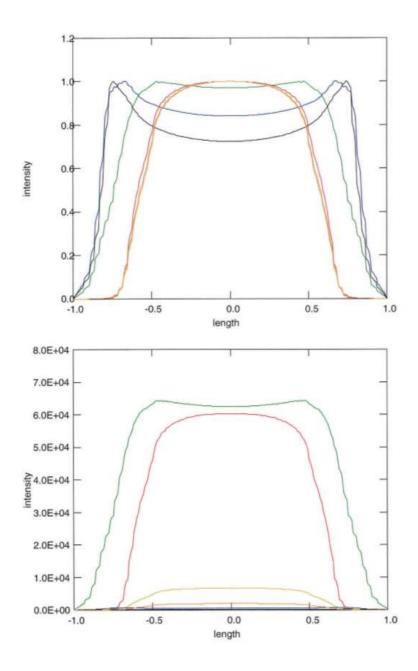


Figure 3: Relative (top) and absolute (bottom, in $erg~cm^{-2}~s^{-1}~sr^{-1}~\mathring{\rm A}^{-1}$) intensities for $L\alpha$ (green), $L\beta$ (blue), $H\alpha$ (red), $L\gamma$ (black), $H\beta$ (yellow), $P\alpha$ (orange) with respect to the position along an axis perpendicular to the cylinder

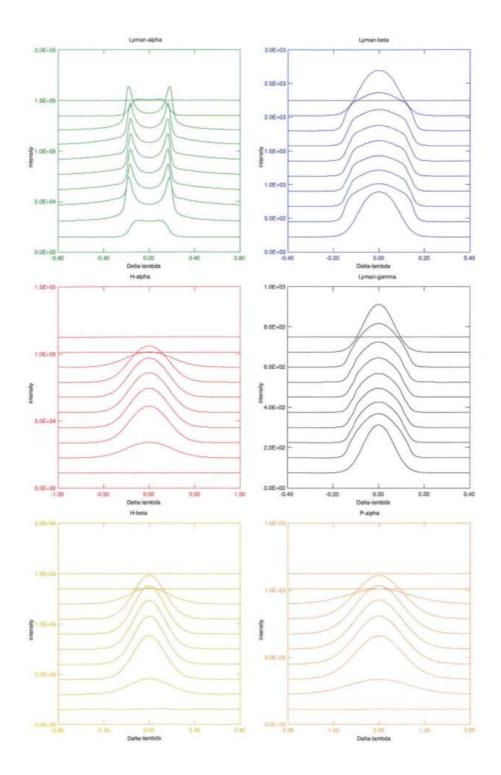


Figure 4: Intensity variations for $L\alpha$ (1215Å), $L\beta$ (1025Å), $H\alpha$ (6564Å), $L\gamma$ (972Å), $H\beta$ (4862Å), $P\alpha$ (18756Å). Abscissae (Å), ordinates ($erg~cm^{-2}~s^{-1}~sr^{-1}~Å^{-1}$)

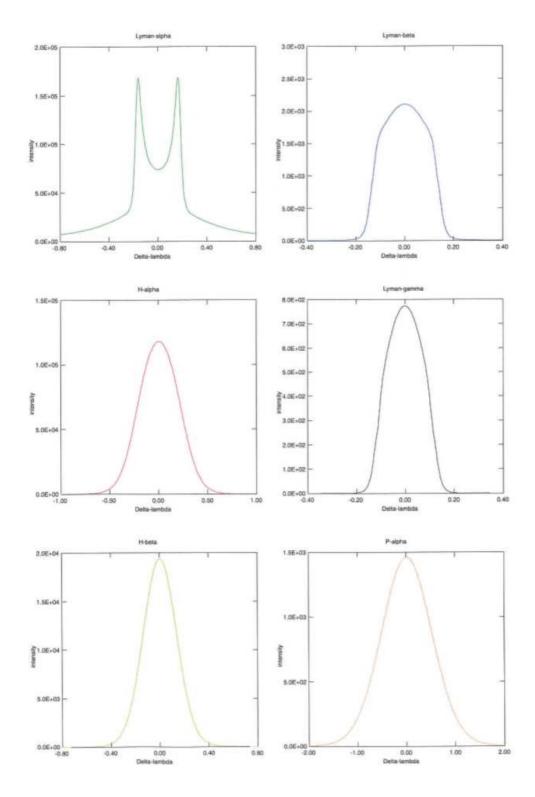


Figure 5: Line profiles for $L\alpha$ (1215Å), $L\beta$ (1025Å), $H\alpha$ (6564Å), $L\gamma$ (972Å), $H\beta$ (4862Å), $P\alpha$ (18756Å). Abscissae (Å), ordinates $(erg~cm^{-2}~s^{-1}~sr^{-1}~Å^{-1})$

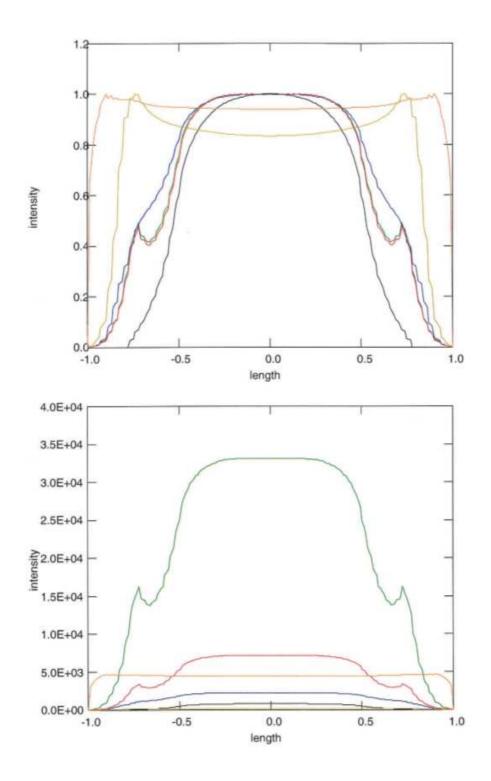


Figure 6: Relative (top) and absolute (bottom, $erg~cm^{-2}~s^{-1}~sr^{-1}~\mathring{\rm A}^{-1}$) intensities for He~I (10830Å, green), He~I (584Å, blue), He~I~D3 (red), He~I (6878Å, black), He~I (537Å, yellow), He~II (304Å, orange) with respect to the position along an axis perpendicular to the cylinder

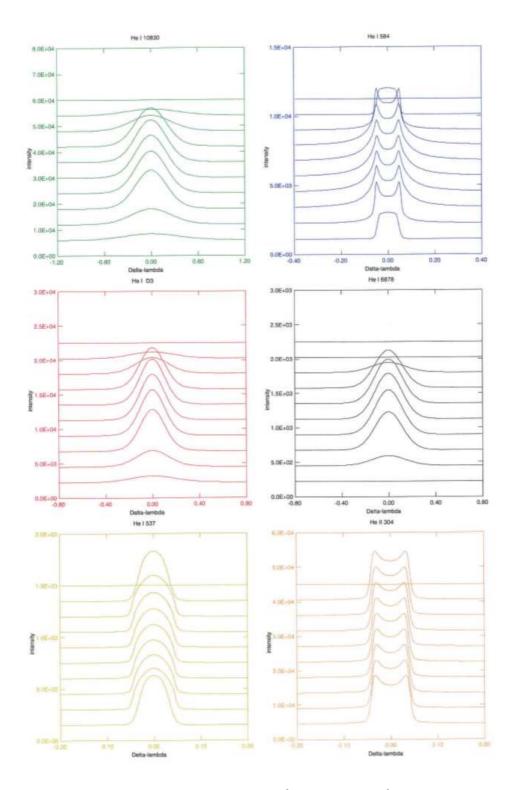


Figure 7: Intensity variations for $He~I~(10830\text{\AA}),~He~I~(584\text{Å}),~He~I~D3~(6564\text{Å}),~He~I~(6878\text{Å}),~He~I~(537\text{Å}),~He~II~(304\text{Å}).$ Abscissae (Å), ordinates $(erg~cm^{-2}~s^{-1}~sr^{-1}~\text{Å}^{-1})$

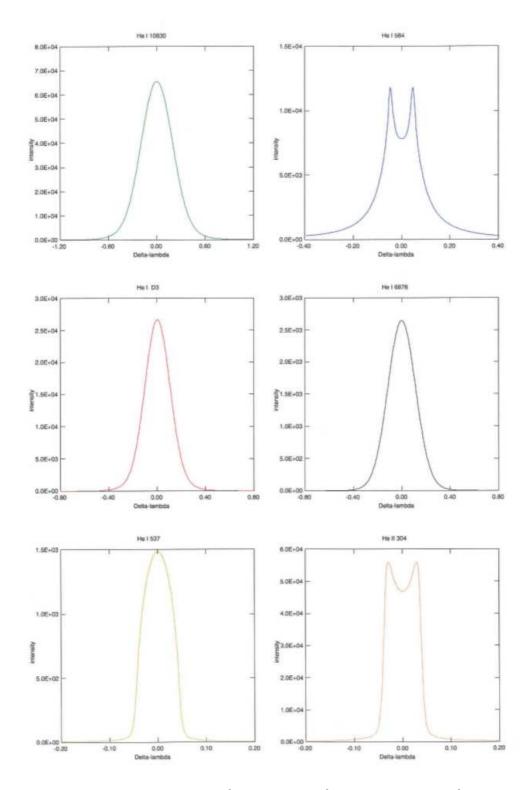


Figure 8: Line profiles for $He~I~(10830\text{\AA}),~He~I~(584\text{Å}),~He~I~D3~(6564\text{Å}),~He~I~(6878\text{Å}),~He~I~(537\text{Å}),~He~II~(304\text{Å}).$ Abscissae (Å), ordinates $(erg~cm^{-2}~s^{-1}~sr^{-1}~\text{Å}^{-1})$

11 Running C2D2E

- Download the package source file C2D2E.tgz from MEDOC website: https://idoc.ias.u-psud.fr/MEDOC/Radiative transfer codes
- Gfortran compiler is required.
- Unpack the package by typing the following linux command: tar -xvzf C2D2E.tgz
- Go to folder C2D2E: cd C2D2E
- The folder contains the following files: helium.com, hydrog.com, intinc.dat, intinc_He.dat, param.com, paramod.dat, c2d2e.f90, visu.f90, makefile
- The file to modify is "paramod.dat"
- Run the code by typing: make ./c2d2e
- The ouput files are: resu2d.dat (emergent intensities for plotting line profiles), fort.20 (electron density), fort.23 (hydrogen level populations), fort.25 (helium level populations)
- The folder **results** contains the output files corresponding to a test case to be able to check if your results are good.
- To visualize resuts, use the following visualization program "visu.f90":
 gfortran -o visu visu.f90
 ./visu
 The ouput file is "cosbou.eps"

12 CPU time

We consider a given model whose parameters vary inside the cylinder (input file "paramod.dat"), defined by the following parameters:

```
IDEMAX = 10, ICRMAX = 4, IVI = 0, TMOD = 0, PGMOD = 0.1, DIAM = 0, ALDEG = 0, HAL = 0 km, ITVAR = 1, R0KM = 500, R1KM = 1000, T0 = 6000, T1 = 10^5, TREF = 10000.
```

On a PC with 4 Intel processors $(2.67 \ GHz)$, the CPU time is $5428.123 \ s$, i.e. 1h30.

13 Acknowledgements

We thank MEDOC directors (Eric Buchlin, present; Frédéric Baudin, former).

28th September 2015 M. C-Y

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