

# USER GUIDE

## HYDR\_NV: Non-LTE radiative transfer in stellar atmospheres

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# 1 Introduction

The HYDR\_NV code was written in Fortran 77 by P. Gouttebroze ([12], [10], [13]) and describes Non-LTE (NLTE) radiative transfer (1D) for hydrogen atom in stellar atmospheres (including solar atmosphere) and in solar prominences (for a more recent prominence version, see PROM7 code and its user guide), using partial frequency redistribution for resonance lines of hydrogen. We solve the equations of radiative transfer, hydrostatic equilibrium, statistical equilibrium of level populations, and ionization equilibrium for the hydrogen atom using an iterative method. HYDR\_NV code is used for comparison with observations.

A stellar atmosphere model is defined by the variation of temperature  $T$  with respect to column-mass  $m$  ( $\text{g}/\text{cm}^2$ ), microturbulent velocity, main element abundance, gravity, star rotation velocity.

There is no incident intensity at the upper limit of the model and we assume that intensity at the lower limit is equal to the Planck function at the temperature considered (LTE).

The original version of HYDR\_NV, adapted to **gfortran** compiler, is available from MEDOC website:

[https://idoc.ias.u-psud.fr/MEDOC/Radiative transfer codes](https://idoc.ias.u-psud.fr/MEDOC/Radiative%20transfer%20codes)

In the following sections, we will explain in details HYDR\_NV code (HYDRogen, Non Vectorized code): modeling, implemented equations, algorithm, numerical methods, etc.

## 2 Description of HYDR\_NV code

*Nature of the physical problem:* NLTE radiative transfer (1D) in stellar atmospheres and solar prominences (HI)

*Method of solution:* Feautrier method with variable Eddington factors

*Other relevant information:*

- Semi-infinite atmosphere models considered here are defined by a variation of temperature  $T$  with respect to column-mass, microturbulent velocity, gravity, main element abundance, star rotation velocity

- Partial frequency redistribution (PRD) is used for resonance lines of hydrogen, while for the other lines and continua complete frequency redistribution (CRD) is used

*Author:* P. Gouttebroze

*Program available from:*

[https://idoc.ias.u-psud.fr/MEDOC/Radiative transfer codes](https://idoc.ias.u-psud.fr/MEDOC/Radiative%20transfer%20codes)

*Computer(s) on which program has been tested:* PC with 4 Intel processors ( 2.67GHz)

*Operating System(s) for which version of program has been tested:* Linux

*Programming language used:* Adapted in Fortran 90 for **gfortran** compiler

*Status:* Stable

*Accessibility:* open (MEDOC)

*No. of code lines in combined program and test deck:*  $\sim 9000$

*Typical running time:*  $< 1$  min for stellar atmosphere model “ALTAIR” and  $< 1$  min for solar atmosphere “VAL3C”

*References:*

- P. Gouttebroze, P. Lemaire, J.-C. Vial and G. Artzner, “The solar hydrogen Lyman- $\beta$  and Lyman- $\alpha$  lines : disk center observations from OSO 8 compared with theoretical profiles”, APJ, 225 : 655-664, 1978
- P. Gouttebroze, P. Heinzel and J.C. Vial, “The hydrogen spectrum of model prominences”, A&A Suppl. Ser. 99, 513-543, 1993

### 3 Algorithm

The figure 1 describes the algorithm of HYDR\_NV code.

The algorithm starts with an initialization of physical parameters for hydrogen:

- Statistical equilibrium (see figure 3) :  $A_{ji}$  (Einstein  $A$  coefficient),  $B_{ij}$  and  $B_{ji}$  (Einstein  $B$  coefficients),  $C_{ij}$  et  $C_{ji}$  (collisional excitation and deexcitation coefficients)
- Boundary conditions
- Radiation temperatures

The aim of HYDR\_NV code is to calculate the electron density, hydrogen level populations, emergent intensities for different values of  $\mu = \cos \theta$  (3 values here: 0.2, 0.6 and 1) where  $\theta$  is the angle between the light ray and the normal to the solar surface.

In the next sections, we consider only semi-infinite atmospheres. For solar prominences, see PROM7 code and its documentation.

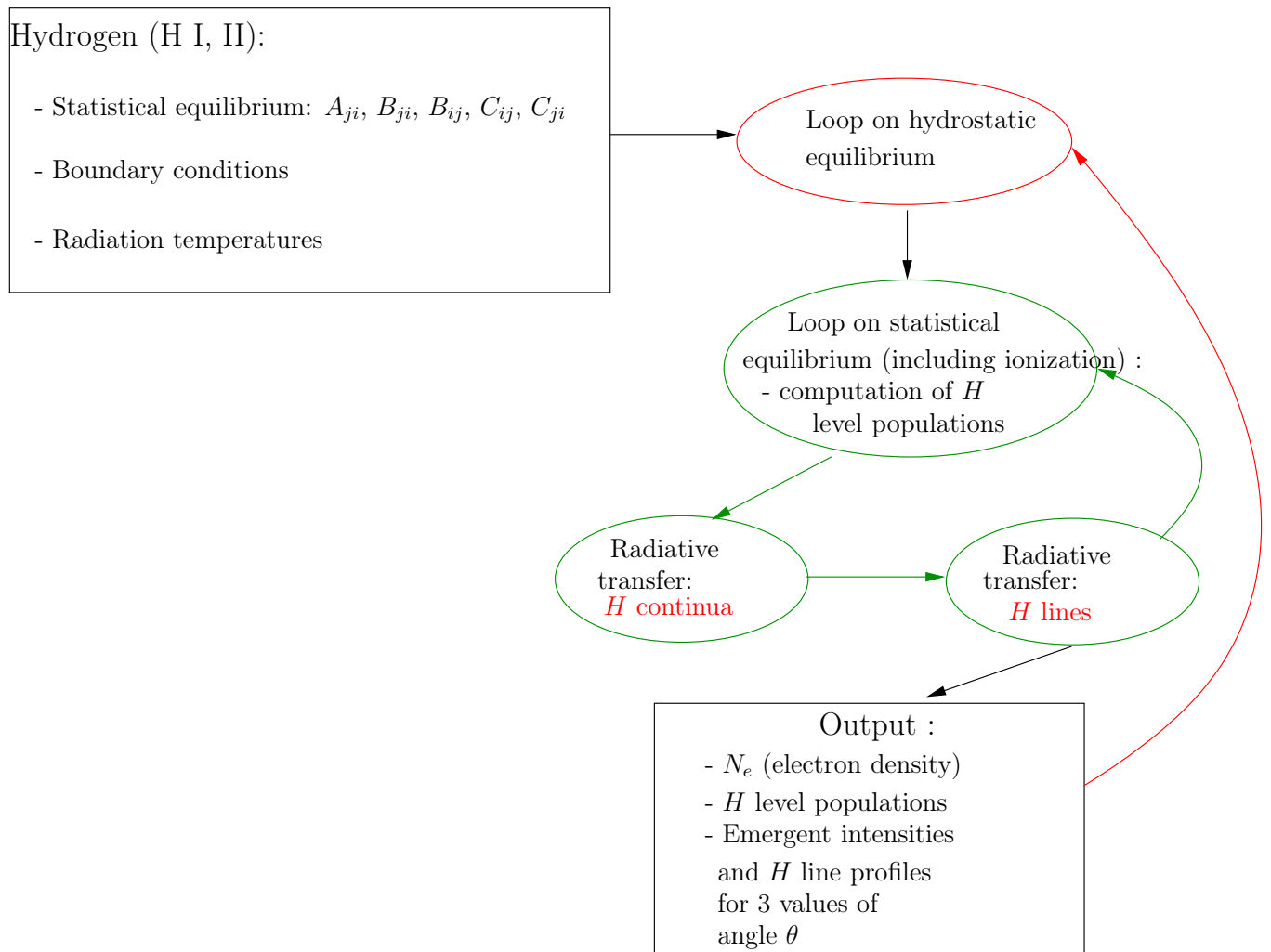


Figure 1: Algorithm of HYDR\_NV code

## 4 Semi-infinite atmosphere modeling

The semi-infinite atmosphere considered here concerns the photosphere and the chromosphere. The layers (meshing) start from the solar corona to photosphere. The layers are spherical but they are considered as plane-parallel slabs.

Figure 2 illustrates this modeling.

## 5 Bound-Free and Bound-Bound transitions

The transitions used in HYDR\_NV code are Bound-Free and Bound-Bound transitions, which are represented in figure 3. Bound-Free transitions are between a bound state  $i$  and a continuum, producing a free electron with energy  $\epsilon$ . It starts from excited states limit, i.e.  $\epsilon = 0$ . Bound-Bound transitions are from level  $i$  to level  $j$ .

## 6 Atomique structure of hydrogen, Johnson model ([16])

The transitions for hydrogen are summarized in Table 1.  $NN$  is the number of energy levels, including the threshold level continuum, bound levels limit ( $\infty$ ).  $NTT = NN \times (NN - 1)/2$  is the total number of transitions,  $NTAC = NN - 1$  is the number of continua (bound-free transitions) and  $NTAR = NTT - NTAC$  is the number of lines (bound-bound transitions). In HYDR\_NV,  $NN = 21$  (20 levels and 1 continuum).

## 7 Population equations and radiative transfer equation

Let's consider level  $j$  in figure 3. At equilibrium, the population equations for this level contain all processes of population and depopulation of level  $j$  to other levels  $i$  which are balanced:

$$N_j \sum_{j \neq i}^n P_{ji} = \sum_{j \neq i}^n N_i P_{ij} \quad (7.1)$$

The depopulation rate  $P_{ji}$  from level  $j$  is written as :

$$P_{ji} = A_{ji} + B_{ji} \bar{J}_{ij} + C_{ji} = R_{ji} + C_{ji} \quad (7.2)$$

where  $C_{ji}$  is the collisional deexcitation rate, proportional to the electron density.  $R_{ji}$  is the radiative depopulation probability either by absorption and by spontaneous ( $A_{ji}$ ) or by stimulated emission.  $A_{ji}$  and  $B_{ji}$  are corresponding to Einstein coefficients.

The population rate  $P_{ij}$  of the level  $j$  can also be written as :

$$P_{ij} = B_{ij} \bar{J}_{ij} + C_{ij} = R_{ij} + C_{ij},$$

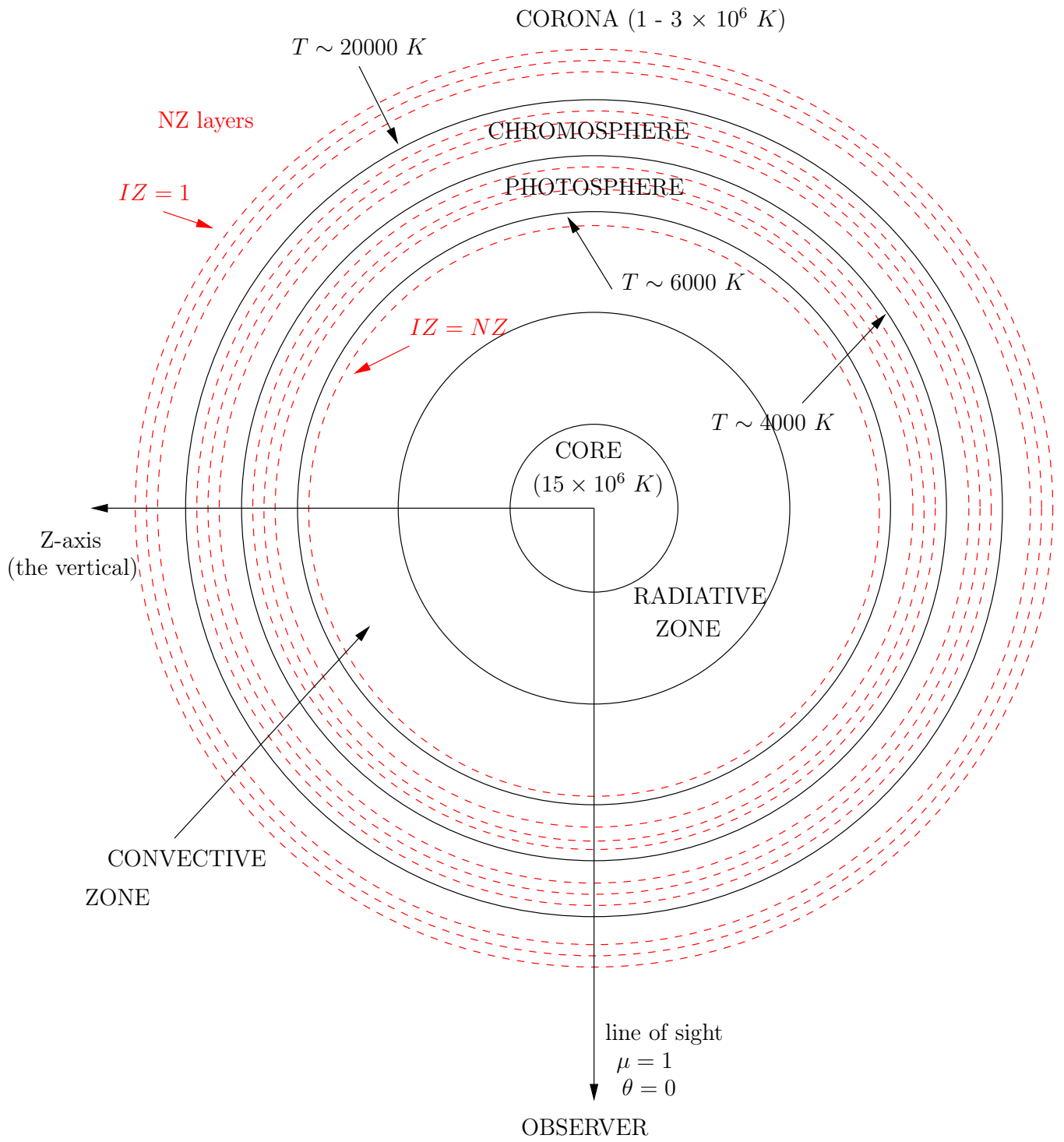


Figure 2: Solar semi-infinite atmosphere model (not to scale)

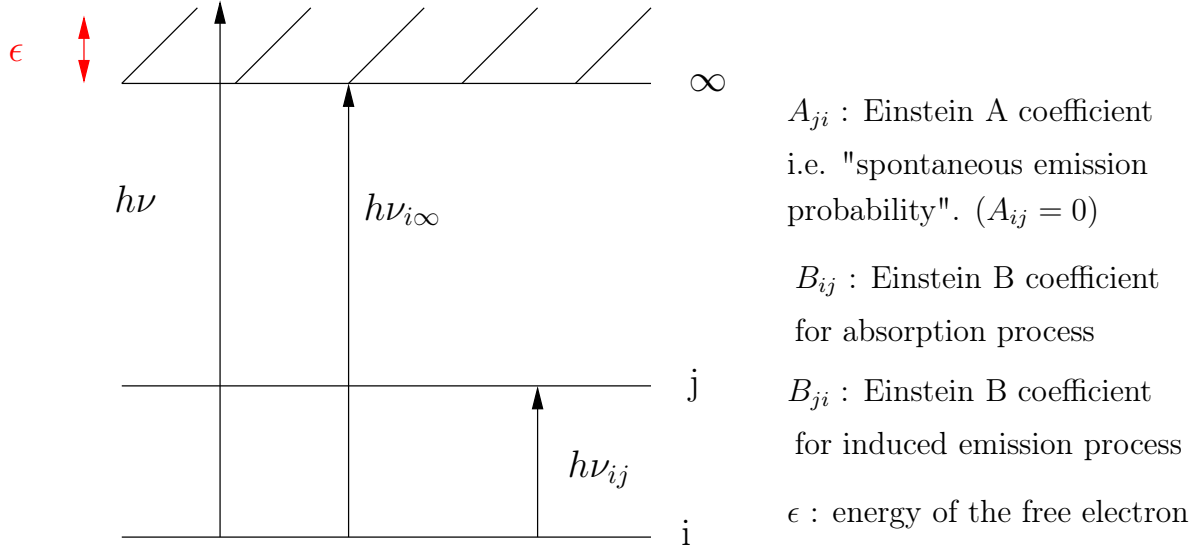


Figure 3: Transition types used in HYDR\_NV

where  $R_{ij}$  and  $C_{ij}$  are similar to  $R_{ji}$  and  $C_{ji}$ , except for spontaneous emission ( $A_{ij} = 0$ ). The expression of  $\bar{J}_{ij}$  is:

$$\bar{J}_{ij} = \int_{-\infty}^{+\infty} J_{\nu} \Phi_{\nu} d\nu \quad (7.3)$$

where  $J_{\nu}$  is mean intensity integrated over direction  $\mu = \cos \theta$ ,  $\theta$  being the angle between the light ray and the normal to the solar surface:

$$J_{\nu} = \frac{1}{2} \int_{-1}^1 I_{\nu}(\mu) d\mu \quad (7.4)$$

$\Phi_{\nu}$  is the normalized profile of the absorption coefficient  $\left( \int \Phi_{\nu} d\nu = 1 \right)$ .

The left-hand side term in the population equations (7.1) represents all the processes which depopulate level  $j$  and the right-hand side term corresponds to all the levels which populate the level  $j$ . And we use the following closure conservation equation:

$$\sum_{j=1}^n N_j = N_T \quad (7.5)$$

where  $N_T$  is the total population of the atomic element.

These population equations (7.1) which contain the mean intensities via  $\bar{J}_{ij}$  (i.e. integrated over the line profile centered in  $\nu_0$ ) are coupled with the radiative transfer equation (7.6).



Transition	Lower level	Upper level	Transition name	Wavelength $\lambda(\text{\AA})$
1	1	2	$Ly - \alpha$	1215
2	1	3	$Ly - \beta$	1025
3	2	3	$H - \alpha$	6564
4	1	4	$Ly - \gamma$	972
5	2	4	$H - \beta$	4862
6	3	4	$Pa - \alpha$	18756
7	1	5	$Ly - \delta$	949
8	2	5	$H - \gamma$	4341
9	3	5	$Pa - \beta$	12821
10	4	5	$Br - \alpha$	6563
11	1	6	$Ly - \epsilon$	973
12	2	6	$H - \delta$	4102
13	3	6	$Pa - \gamma$	10935
14	4	6	$Br - \beta$	4861
15	5	6	$Pf - \alpha$	7460
16	1	7	$Ly - 6$	931
17	2	7	$H - \epsilon$	3971
...	...	...	...	...
$NTAR + 1$	1	$NN$	Lyman Continu	911
$NTAR + 2$	2	$NN$	Balmer continu	3645
...	...	...	...	...
$NTT =$	$NN - 1$	$NN$		

Table 1: Main transitions for hydrogen

The intensity in the radiative transition is obtained by solving the transfer equation:

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu \quad (7.6)$$

$\tau_\nu$  is the optical depth at frequency  $\nu$ , and  $S_\nu$  is the total source function defined by:

$$S_\nu = \frac{\epsilon_\nu}{\kappa_\nu},$$

where  $\kappa_\nu$  is the absorption coefficient and  $\epsilon_\nu$  the emission coefficient.

Thus, we obtain a coupled equation system which is solved by numerical methods such as the Feautrier method ([14]). For more details, see the next two sections.

## 8 Formalism used for radiative transfer

The radiative transfer equations in the lines and continua of hydrogen are solved by a finite-difference method (Feautrier method) with variable Eddington factors. The other equations,

namely that of statistical equilibrium of level populations, ionization and pressure equilibria, are solved by iteration. This is basically equivalent to a two-level atom model.

At the beginning, the intensities inside the slab are set by assuming that the medium is optically thin in all transitions (i.e. without radiative transfer). With these intensities, we compute the radiative transition rates. We start from an arbitrary electron density (half of the hydrogen density) to determine the collisional rates. Then, we solve the statistical equilibrium equations to determine hydrogen level populations and obtain new electron densities. The hydrogen densities are adjusted in order to satisfy the pressure equilibrium condition. After a few iterations, we obtain hydrogen and electron densities which are consistent with the intensities.

We compute the optical depths and solve the radiative transfer equations for the continua and the lines which are optically thick. The incident intensities are used as boundary conditions. The Lyman- $\alpha$  and  $\beta$  lines are treated according to standard partial redistribution, while complete redistribution is assumed for other lines. These computations produce a new set of intensities, which in turn yields new radiative rates, and new populations through the statistical equilibrium equations. In practice, “net radiative rates” (not explained here) in lines are used to control the convergence, with the help of relaxation parameters. This process is applied many times before new hydrogen and electron densities are computed to satisfy pressure and ionization equilibria.

The process stops when some fixed convergence criteria are satisfied both for net radiative rates in lines and radiation temperatures in continua.

## 9 Subroutine descriptions

HYDR\_NV calls INIT subroutine (code options, see paragraph 9.1), INIMET subroutine (metal abundances relative to hydrogen) and the main subroutine EXEATM (computations in stellar atmospheres). Each subroutine will be explained in details in paragraph 9.2.

**Input** file for HYDR\_NV: fort.3 (solar or stellar atmosphere model). The first line of fort.3 contains 4 values: ILG (gravity acceleration in real value if ILG=0 or in logarithmic value if ILG=1), GGG (gravity acceleration value in  $\text{cm.s}^{-1}$  units), ILM (column-mass in real values if ILM=0 or in logarithmic values if ILM=1), VISINI (star rotation velocity). The 4 following data columns represent column-mass ( $\text{g.cm}^{-2}$ ), temperature ( $^{\circ}\text{K}$ ), microturbulent velocity ( $\text{km.s}^{-1}$ ) and index specifying the reference altitude (index=1 means there is a reference altitude, index=0 means there is no reference altitude and index=-1 means the end of the input file).

**Output** files for HYDR\_NV:

- fort.21 : intensities for 3 directions  $\mu = \cos \theta$  and for each line transition
- fort.22 :  $L\alpha$  intensities versus frequency and emission angle  $\mu$
- fort.24 : atmospheric parameters, hydrogen density, electron density, radiation temperatures, H level populations
- fort.28 : mean intensities (to determine photoionization rates of different elements)
- fort.8 : iteration summary in CPU time

## 9.1 Set of variables used in EXEATM subroutine

- IVERT = -1 (solar or stellar atmosphere case), IVERT = 0 (prominence case, vertical slab)
- NZ: total number of meshes
- NFR: number of frequencies in a line
- NMU: number of directions  $\mu = \cos \theta$ , where  $\theta$  is the angle between the light ray and the normal to the solar surface
- NPSOR: number of points for the visualization of line profiles
- NFRC: number of frequencies in a continuum
- NTAB: size of the temperature array  $TAB$  (for the computation of collision rate coefficients)
- NINF: transition lower level
- NSUP: transition upper level
- ITP: to indicate if the transition is allowed or forbidden
- IOPRN = 1: complete redistribution (CRD), IOPRN = 3: partial redistribution (PRD), IOPRN = 4: PRD with coherent coefficient depending on frequency (particular case for  $L\gamma \alpha$ )
- IOPERA = 1: line profiles are printed, IOPERA = 0: line profiles are not printed. IOPERA is linked to DL2 (see below)
- DL1: x-axis (first wavelengths) for line profiles (visualization)
- DL2: y-axis (last wavelengths) for line profiles (visualization)

- IOPMRU = 1: only one redistribution matrix is computed for each transition (the same matrix is used for all meshes), IOPMRU = 0: all the redistribution matrices are computed for each transition (the same matrix is used for all meshes)
- EPS and BEN: line coupling coefficients
- CSI0 and ETA0: parameters for continua which are equivalent to EPS and EPSxBEN for lines
- GIBAR: mean intensity weighted by absorption profile
- IOPECO: equivalent to IOPERA for continua
- IOPFEV: option for computing variable Eddington factors
- IPROF = 1: optical depths are printed. If not, IPROF = 0
- IOPMIC = 1: microturbulent pressure is taken account. If not, IOPMIC = 0
- INIDLD: option for initializing automatically Doppler widths from temperature of reference TERE (for graphic representation)
- IOPCAC = 0: without continuum absorption, IOPCAC = 3: continuum absorption is included in computations
- IOPEAC: prints continuum absorption
- IOPTTC: prints continuum transition rates
- PMU: values of  $\mu$  (line profiles for  $\mu = 1$  is plotted in plain line, for  $\mu = 0.6$  in dashed line, and for  $\mu = 0.2$  in dotted line)
- IVTR: visualization index for radiative transfer
- HNH: hydrogen density (number of atoms per unit volume)
- HNE: electron density
- XM: column-mass
- ETOT: total energy emitted by the structure in a line
- RDMAT: redistribution matrix
- AJI: Einstein A coefficient i.e. spontaneous emission
- BJI, BIJ: Einstein B coefficients for absorption and induced emission processes
- CIJ, CJI: collisional excitation/deexcitation coefficients

- CAC: continuum absorption coefficient
- TR: radiation temperature for bound-free transitions
- FADI: dilution factor for lines and continua
- FEVK and FEVL, FKTC and FLTC: variable Eddington factors (resp. lines and continua)
- HIEMC: intensity emitted in the continuum
- RIK, RKI: radiative transition rates
- FIIR, FISR: lower and upper incident line flux
- FRR: frequency in the line
- IOPTR (for continua)= 0: radiation temperature is local electron temperature, IOPTR=1: junction with chromosphere, IOPTR=2: junction with photosphere
- TROM (for continua): radiation temperature corresponding to the optically thin part of the atmosphere
- XFR and XFRC: division model in frequency for lines and continua
- NTR and NTC: discrete and bound-free transition numbers
- ICTR and ICTC: control index for lines and continua
- FRN: level frequency
- BRN and BRP: net radiative bracket before and after computations (they are used to control the convergence)
- SC: continuum source function
- STR: total source function (for line and continuum)
- COHER: coherent coefficient
- OIS: collisional ionization rate coefficient

## 9.2 Hydrogen modeling (Johnson model, [16] )

Hydrogen modeling (EXEATM subroutine) is divided in several parts:

- INIATM3: reading of the input file “fort.3” (atmosphere model parameters)
- INIFEV: initialization of Eddington factors (lines and continua)
- JEUXDO: hydrogen atomic data
- COLIM3: boundary conditions (lines and continua)
- INITR3: initialization of radiation temperatures  $TR$  for hydrogen (corresponding to the radiation of a black body) inside the slab
- PENDULE: gives CPU time
- Loop on hydrostatic equilibrium:  
*do ITG = 1, ITGMAX:*
  - HYESEH: hydrostatic equilibrium + ionization equilibrium
  - ABSCON (if  $IOPCAC = 3$ ): computation of continuum absorption due to different elements (defined in INIMET subroutine)
  - *do IT = 1, NTAR:* loop on discrete transitions  
RPCDEF: computation of partial and complete redistribution matrices  
*enddo*
  - If  $IOPFEV \geq 3$ , Eddington factors for lines and continua are printed via ECRFEV subroutine
  - *do itac = 1, NTAC:* loop on continua  
ECRFEV  
*enddo*  
*endif(for IOPFEV  $\geq$  3)*
  - ESTR6: Solution of statistical equilibrium equations for hydrogen excited level populations ( $HN$ )
  - NOVTER: Update of radiation temperature  $TR$
  - PENDULE*enddo* (for  $ITG$ )
- If  $IOPFEV \geq 2$ :
  - *do IT = 1, NTAR:* loop on discrete transitions  
ECRFEV  
*enddo*

- *do itac = 1, NTAC*: loop on continua  
ECRFEV  
*enddo*
- endif(for IOPFEV ≥ 2)*
- TESTH6: test (we compute once again the quantities)
  - IMTOCS (if *IOPTOC = 1*): prints continuum optical depths versus altitude for different wavelengths
  - CSHYDR: computation of hydrogen spectrum (UV,  $H\alpha$ ,  $L\alpha$ )
  - CATFIC (if *ICATA = 1*): prints (in output file fort.24) atmospheric parameters, hydrogen density, electron density, radiation temperatures, hydrogen level populations
  - CATF27 (if *ICATA = -1*): is a variant of CATFIC adapted to hydrodynamic problems, prints parameters in output file “fort.27”
  - IMPIMO (if *ICATA = 2*): computes mean intensities (to determine photoionization rates for different elements) and prints them in output file “fort.28”

Subroutines called above are explained in the next paragraph.

## INIT

We define here the options of the code for calculations (see paragraph 9.1 for option listing).

## INIMET

INIMET computes total weight  $C_1$  of elements and total abundance  $C_2$  with respect to hydrogen of the  $NEL = 20$  following elements: He, C, N, O, Ne, Na, Mg, Al, Si, P, S, Ar, K, Ca, Cr, Mn, Fe, CO, Ni, H.

Outputs:  $C_1$  et  $C_2$

## INIATM3

INIATM3 reads the input file “fort.3”. After reading the model data (column-mass  $m$ , temperature  $T$  and microturbulent velocity  $V_T$ ), we compute the number of layers (meshes)  $NZ$  to be considered, electron temperature  $T_e$ , microturbulent velocity  $V_T$  with respect to  $T_e$ .

## INIFEV

Initialization of line (*FEVK* and *FEVL*) and continua (*FKTC* et *FLTC*) Eddington factors at each frequency and for each mesh.

Outputs: *FEVK*, *FEVL*, *FKTC* and *FLTC*

## JEUXDO

Computation of hydrogen atomic parameters for statistical equilibrium: Einstein *A* and *B* coefficients for photons (*AJI*, *BIJ*, *BJI*: independent of electron temperature), collisional excitation and ionization rate coefficients (*OIS* in HYDR\_NV code) for electrons, called  $S_e$  and  $S_i$  in the formulas below. These coefficients are independent of electron temperature. The model used for hydrogen is L.C. Johnson model ([16]).

*AJI* is obtained by AEMS function (in HYDR\_NV code) which is calculated from “Gaunt factors” (Table 1 in [16]).

*BJI* is obtained from *AJI*, *BIJ* is such that  $\omega_i B_{ij} = \omega_j B_{ji}$ , where  $\omega_i$  is the statistical weight associated to level *i*.

## Computation of collisional excitation rate coefficients (CECH function in HYDR\_NV code):

Let *n* and *n'* be two levels ( $n < n'$ ). The excitation rate coefficient is given by the following formula (Johnson, [16]), implemented in HYDR\_NV code:

$$S_e(n, n') = (8kT/\pi m)^{1/2} \frac{2n^2}{x} \pi a_0^2 y^2 \left( A_{nn'} \left[ \left( \frac{1}{y} + \frac{1}{2} \right) E_1(y) - \left( \frac{1}{z} + \frac{1}{2} \right) E_1(z) \right] \right) \\ + (8kT/\pi m)^{1/2} \frac{2n^2}{x} \pi a_0^2 y^2 \left( \left[ B_{nn'} - A_{nn'} \ln \frac{2n^2}{x} \right] \left[ \frac{1}{y} E_2(y) - \frac{1}{z} E_2(z) \right] \right) \quad (9.1)$$

$E_i(z) = \int_1^\infty e^{-zt} t^{-i} dt$  is called Exponential integral of order *i* ( $i = 0, 1, 2, \dots$ ).

*m* denotes the electron mass and  $a_0 = 0.5292 \cdot 10^{-8} \text{ cm}$  the Bohr radius.



Here and below,  $E_n$  denotes level energy of  $n$  for hydrogen. We obtain:

$$\begin{aligned}
y &= (E_{n'} - E_n)/kT, \\
z &= r_{nn'} + y, \\
x &= 1 - (n/n')^2, \\
B_{nn'} &= \frac{4n^4}{n'^3} x^{-2} \left(1 + \frac{4}{3}x^{-1} + b_n x^{-2}\right), \\
b_n &= n^{-1}(4 - 18.63n^{-1} + 36.24n^{-2} - 28.09n^{-3}), \quad n \geq 2, \\
A_{nn'} &= 2n^2 x^{-1} f_{nn'}, \\
f_{nn'} &= \frac{32}{3\sqrt{3}} \frac{n}{\pi n'^3} x^{-3} g(n, x), \\
g(n, x) &= g_0(n) + g_1(n) x^{-1} + g_2(n) x^{-2}, \\
r_{nn'} &= r_n x, \\
r_n &= 1.94n^{-1.57}
\end{aligned}$$

where  $g_0$ ,  $g_1$ ,  $g_2$ : Gaunt factors for pour Bound-Free transitions. They are given in Table 2 (Table 1 in [16]) :

	$n = 1$	$n = 2$	$n \geq 3$
$g_0(n)$	1.11330	1.0785	$0.9935 + 0.2328n^{-1} - 0.1296n^{-2}$
$g_1(n)$	-0.4059	-0.2319	$-n^{-1}(0.6282 - 0.5598n^{-1} + 0.5299n^{-2})$
$g_2(n)$	0.07014	0.02947	$n^{-2}(0.3887 - 1.181n^{-1} + 1.470n^{-2})$

Table 2: Gaunt factors

### Computation of collisional ionization rate coefficients (CICH function in HYDR\_NV code):

Let us consider here level  $n$ . Ionization rate coefficient is given by the following formula (Johnson, [16]) implemented in HYDR\_NV code:

$$\begin{aligned}
S_i(n) &= (8kT/\pi m)^{1/2} 2n^2 \pi a_0^2 y_n^2 A_n \left[ \frac{1}{y_n} E_1(y) - \frac{1}{z_n} E_1(z_n) \right] \\
&+ (8kT/\pi m)^{1/2} 2n^2 \pi a_0^2 y_n^2 (B_n - A_n \ln 2n^2) [\zeta(y_n) - \zeta(z_n)],
\end{aligned} \tag{9.2}$$

where  $m$  is the electron mass and

$$\begin{aligned}
y_n &= I_n/kT, \\
z_n &= r_n + I_n/kT,
\end{aligned}$$

$$\zeta(t) = E_0(t) - 2E_1(t) + E_2(t)$$

where  $I_n$  is ionization energy of level  $n$  for hydrogen.

Outputs:  $AJI$ ,  $BIJ$ ,  $BJI$ ,  $OIS$ ,  $DLDFEF$  in frequency and  $DNDREF$  in wavelength (Doppler width for Bound-Free transitions).

### **COLIM3**

Boundary conditions (hydrogen) for semi-infinite atmosphere concerning Bound-Free and Bound-Bound transitions.

For lines: lower boundary condition  $FIIR$  is equal to Planck function. Upper boundary condition  $FISR$  is null.

For continua: lower boundary condition  $FIIC$  is equal to Planck function. Upper boundary condition  $FISC$  is null.

Outputs :  $FIIR$  and  $FISR$  (for lines),  $FIIC$  et  $FISC$  (for continua).

### **INITR3**

Initialization of radiation temperature ( $TR$ ) corresponding to photoionization rates in hydrogen Bound-Free transitions, from incident radiation temperature and dilution factor  $FADIC$ . We assume at the beginning that the medium is optically thin in all transitions. The photoionization cross-section  $SEP$  for hydrogen is obtained from Gaunt factors ([8]). Radiation temperature changes during the iterative process.

Inputs:  $NZ$ ,  $NN$ ,  $NTT$ ,  $NTAC$ ,  $NFRC$ ,  $NTC$ ,  $NINF$ ,  $FRN$ ,  $XFRC$ ,  $FADI$ ,  $RIK$ ,  $TE$

Output:  $TR$

### **HYESEH**

Solution of statistical equilibrium and ionization for given temperatures and pressures: hydrogen level populations and electron density are computed.

The 5 following subroutines are called in HYESEH:

**EQHYST**: computation of hydrostatic equilibrium.

**TXCOLL**: collisional transition probabilities ( $CIJ$  et  $CJI$ ) are computed by using electron density.

**INIRIK**: initialization of photoionization probabilities (for hydrogen) from radiation temperatures.

**EQST**: solution of statistical equations for hydrogen level populations by iteration. We obtain a linear system to solve. Output :  $HN$  for each level and mesh.

**SAHARA**: computation of electron density  $HNE$  and density of the other elements

*HNION*. Saha law at LTE gives the ionization degree of each ion. We use here *HNH* (and not *HN*).

Inputs pour HYESEH: *NZ, NN, NTT, NTAR, NTAC, NFRC, C1, C2, ITP, NINF, NSUP, NTC, FRN, POM, XFRC, Z, XM, TE, VT, HNH, HNE, HN, NTAB, TAB*

Outputs pour HYESEH: *CIJ, CJI, RIK, RKI, HNE, HNION*

## **ABSCON**

ABSCON is called when  $IOPCAC = 3$  (radiative transfer doesn't take into account this effect). ABSCON computes continuum absorption in the neighbourhood of lines under consideration and uses PROFSC subroutine.

**PROFSC:** computes additional optical depth *TAUC* (due to continuum absorption process), continuous source functions *FSC*, coherent scattering ratios *CSCAT* for a given frequency *FREQ*, and continuous absorption coefficient *CABCO*.

To compute *CABCO*, we need continuum absorption (LTE) by hydrogen negative ions, aluminium, hydrogen (photoionization cross-section *SEPION* is computed from Gaunt factors ([8])), carbon, magnesium, silicium and iron.

Input: *IVISU*

Outputs for ABSCON: *TAUC, FSC, CSCAT, CABCO*

## **RPCDEF (for resonance lines)**

Hydrogen partial redistribution. Broadening constants for lines (natural and collisional) are computed.

RPCDEF uses the following subroutines:

**ELCOH:** computation of collisional broadening *DFRCO* for hydrogen lines

**COHEVA:** computation of coherent coefficient for different frequencies of *Ly*  $\alpha$  line (hydrogen)

**MARALA:** computation of the redistribution matrix for *Ly*  $\alpha$  line

**MAREDI:** computation of the redistribution matrix for the other lines (case of a resonance line with a frequency-independent coherence coefficient)

Inputs for RPCDEH: *ITAR, IVISU*

Output for RPCDEF: *RDMAT* (redistribution matrix)

## **ECRFEV**

Prints variable Eddington factors *FEVK*

## ESTR6

- Computation of hydrogen level populations (EQST subroutine)
- radiative transfer: NLTEBF subroutine (NLTE computation of radiation field and radiative transition probabilities for hydrogen continua), RPRAIE subroutine (radiative transfer for lines) and RAOPMI subroutine (radiative transfer for optically thin lines)
- computation of radiative transition rates for continua (NOVRIK subroutine) and for lines (NOVBRN subroutine)

Input: List of “common” (common variables)

Output for ESTR6: *RIK* (radiative transition rates for continua), *BRN* (Net radiative bracket), *HN* (hydrogen level populations)

## NOVTER

Update of radiation temperature for hydrogen Bound-Free transitions for next iteration *ITG*

Input: List of “common” (common variables)

Output for NOVTER: *TR*

## TESTH6

Global test: we compute once again all the quantities. We call again the following subroutines: EXOATM (prints atmospheric parameters i.e. pressure, altitude, electron temperature, hydrogen density, etc), ABSCON (computation of continuum absorption due to different elements), ECRITAU (prints collisional transition rates and net radiative brackets), EQST (computation of hydrogen level populations), ECRIPOP (prints hydrogen level populations), RPCDEF (partial redistribution for hydrogen resonance lines), RPRAIE (radiative transfer for lines), EXOTDE (prints optical depth, Doppler width, etc), PROEM1 (computation of line profiles for  $\theta = 90^\circ$ ,  $\theta$  being the angle between the light ray and the normal to the solar surface), PROEM2 (computes emergent profile for different values of angle  $\theta$  and prints them in output file “fort.21”), NLTEBF (computation NLTE of radiation fields and radiative transition probabilities for hydrogen continua), EXOTCO (prints emergent intensities for different directions), EXPROB (prints transition probabilities for lines and continua), COMPAR (comparison between *EMIN*, *EMAX*, *EQM*, and *EQMR*), COMPAC (comparison between *EMIN* and *EMAX*).

## IMTOCS

IMTOCS prints continuum optical depths with respect to altitude for different wavelengths.

## CATFIC

CATFIC prints in output file “fort.24” atmospheric parameters, electron density, level populations, radiation temperature and hydrogen density.

## 10 Results for hydrogen lines

### 10.1 Solar atmosphere (VAL3C)

We consider here the solar atmosphere model VAL3C ([24]).

The half-profiles are given for 3 values of  $\mu = \cos \theta$ . Emergent intensity  $I_\nu$  is in  $\text{erg cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{Hz}^{-1}$  units.

The half-profile for  $\mu = 1$  is plotted by a plain line. The half-profile for  $\mu = 0.6$  is plotted by a dashed line and the half-profile for  $\mu = 0.2$  by dotted line.

The following figures for hydrogen lines are stored in the folder **results** when you download the package source file HYDR\_NV.tgz from MEDOC website.

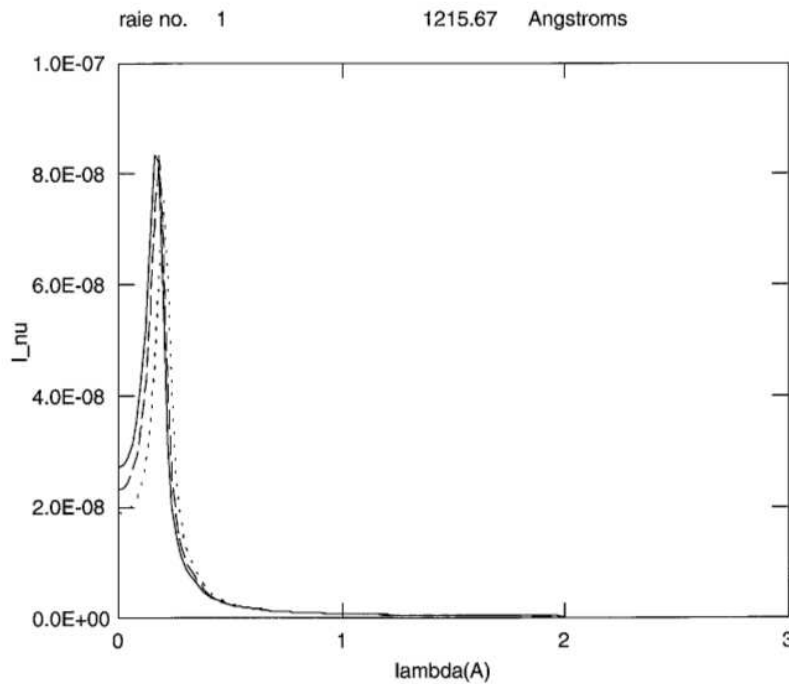


Figure 4: Half-profile of  $L\alpha$  line at  $1215\text{\AA}$

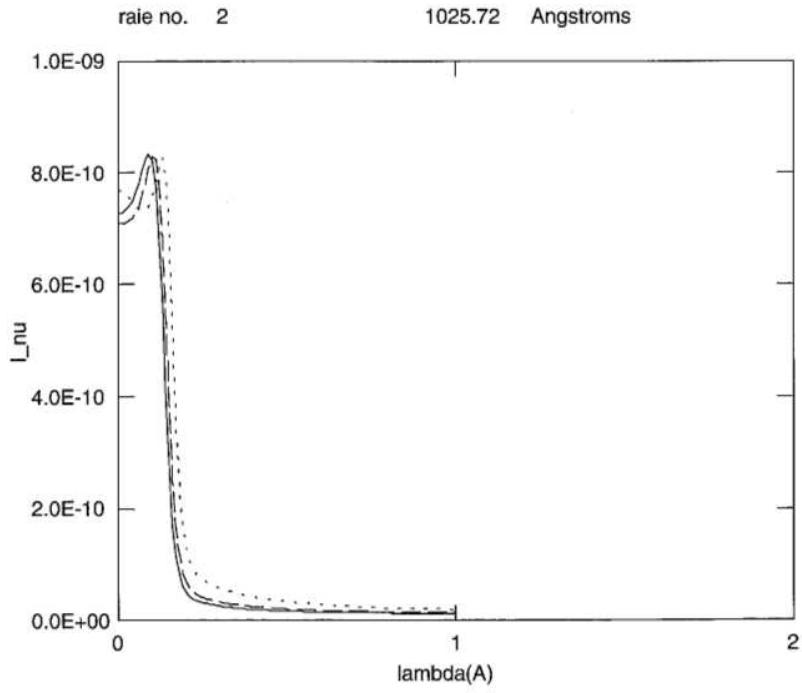


Figure 5: Half-profile of  $L\beta$  line at 1025Å

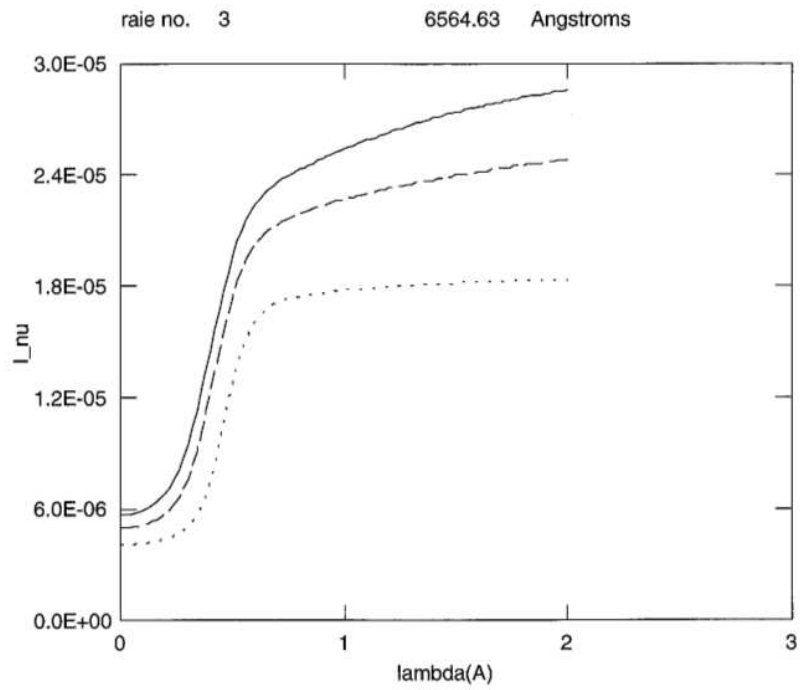


Figure 6: Half-profile of  $H\alpha$  line at 6564Å

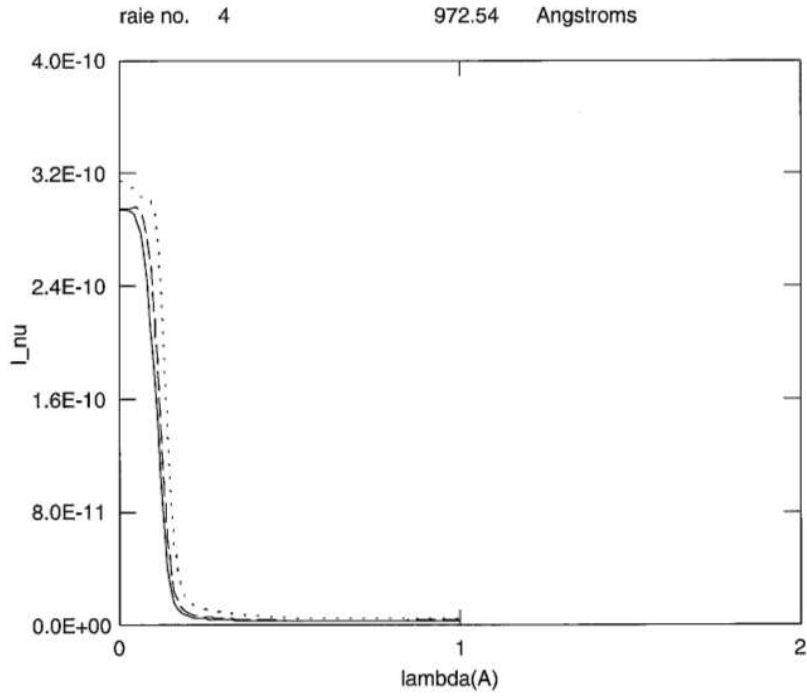


Figure 7: Half-profile of  $L\gamma$  line at  $972\text{\AA}$

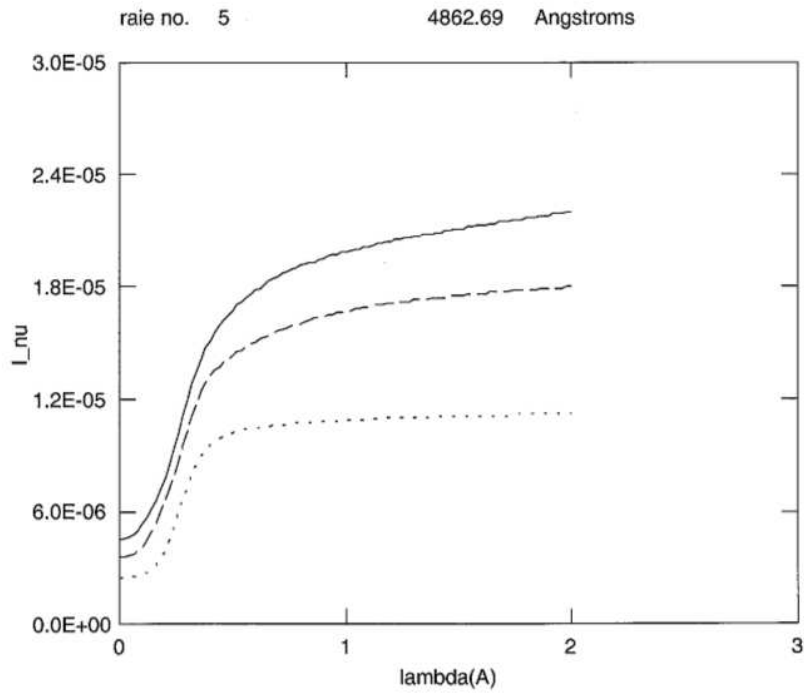


Figure 8: Half-profile of  $H\beta$  line at  $4862\text{\AA}$

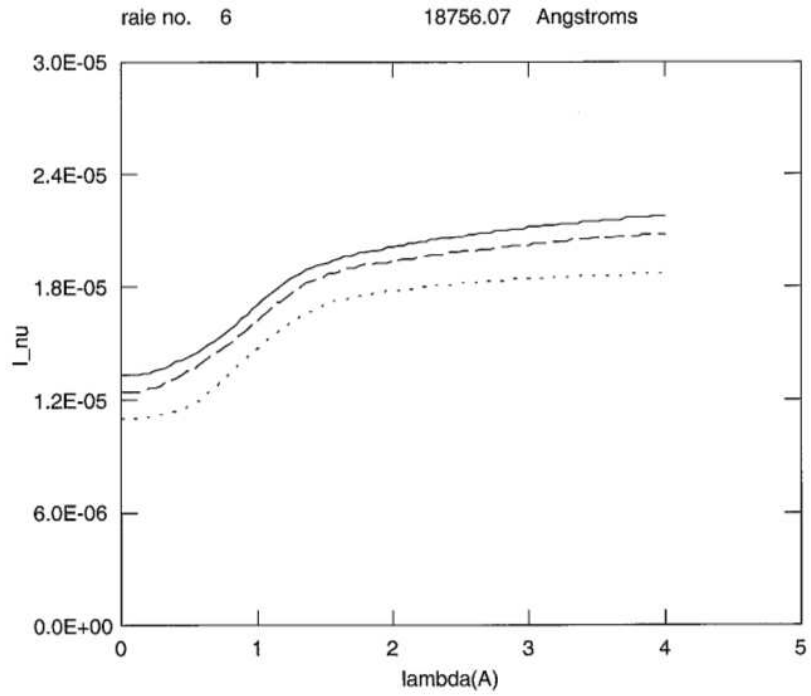


Figure 9: Half-profile of  $Pa - \alpha$  line at  $18756 \text{ \AA}$

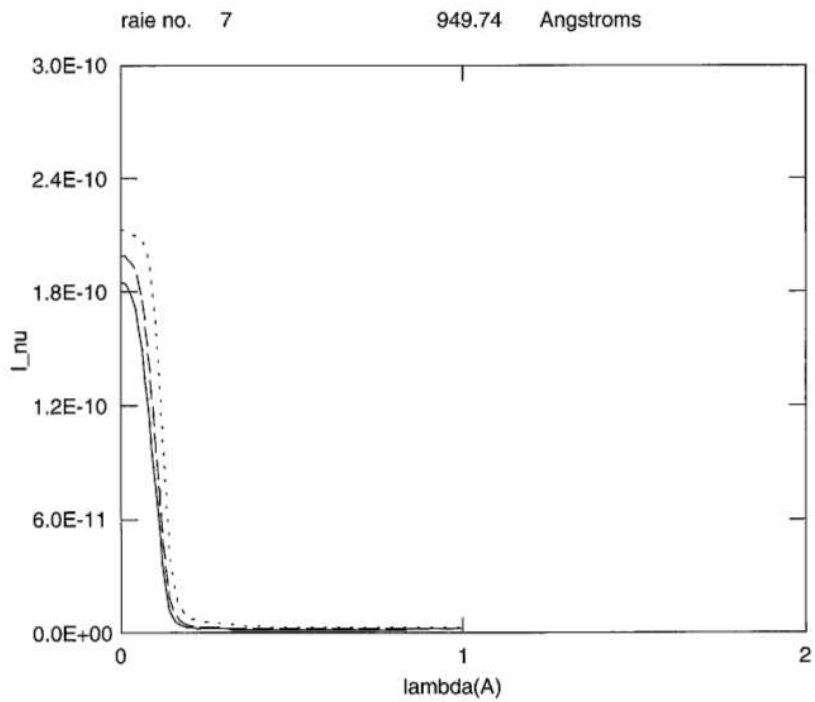


Figure 10: Half-profile of  $L\delta$  line at  $949 \text{ \AA}$



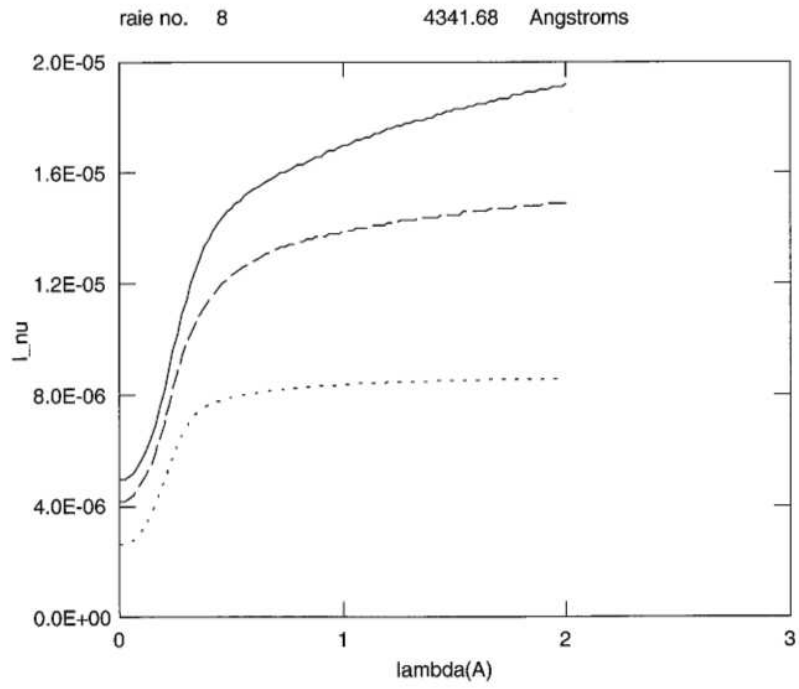


Figure 11: Half-profile of  $H\gamma$  line at 4341Å

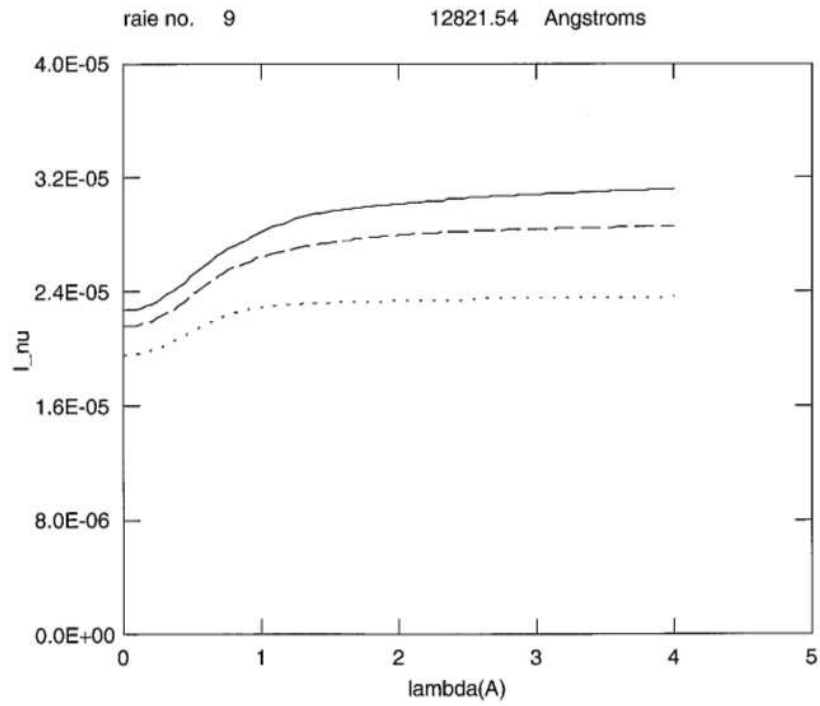


Figure 12: Half-profile of  $Pa - \beta$  line at 12821Å

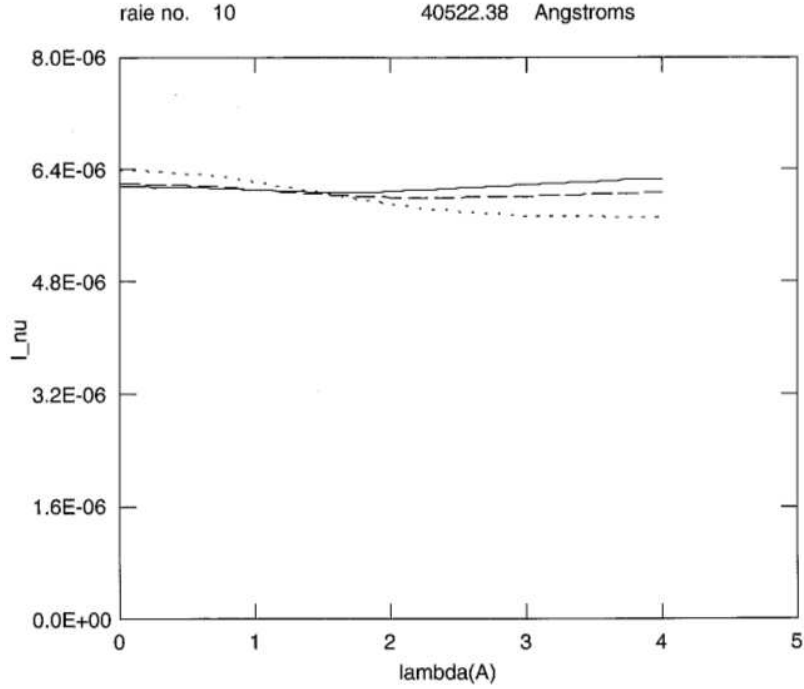


Figure 13: Half-profile of  $Br - \alpha$  line at  $40522\text{\AA}$

## 10.2 Stellar atmosphere (Altair)

We consider here ALTAIR atmosphere model.

The half-profiles are given for 3 values of  $\mu = \cos \theta$ . Emergent intensity  $I_\nu$  is in  $\text{erg cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{Hz}^{-1}$  units.

The half-profile for  $\mu = 1$  is plotted by a plain line. The half-profile for  $\mu = 0.6$  is plotted by a dashed line and the half-profile for  $\mu = 0.2$  by dotted line.

The following figures for hydrogen lines are stored in the folder **results** when you download the package source file `HYDR_NV.tgz` from MEDOC website.

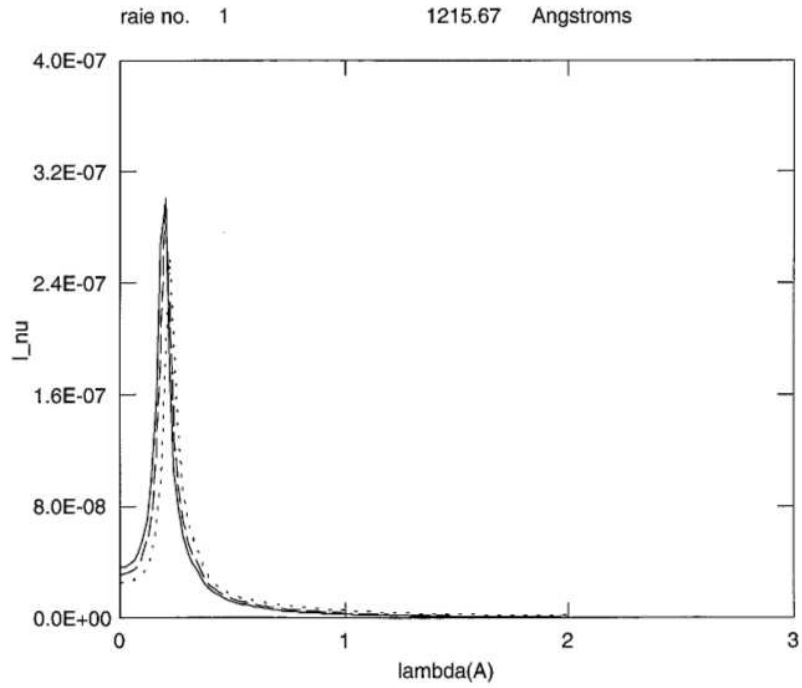


Figure 14: Half-profile of  $L\alpha$  line at 1215Å

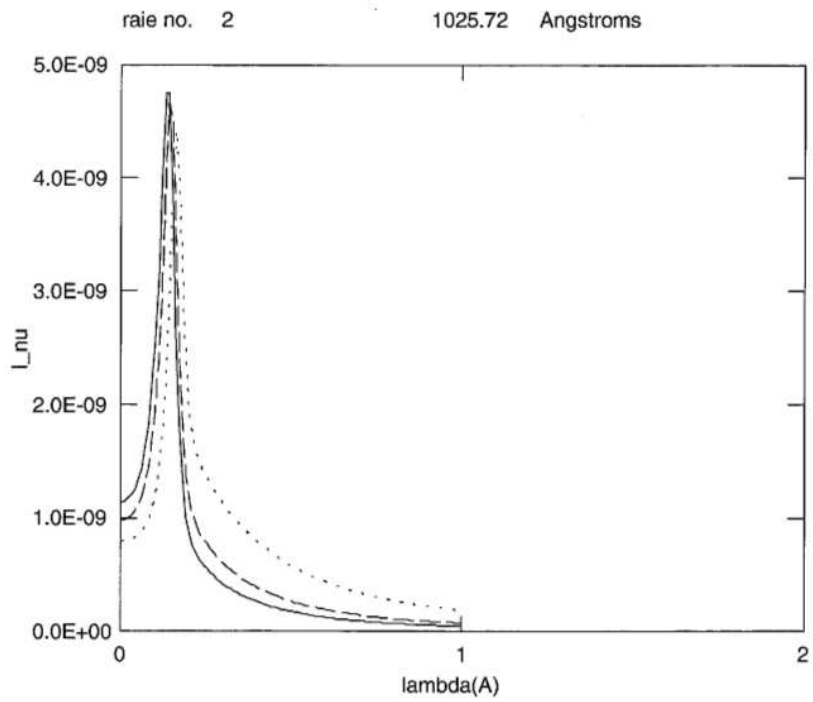


Figure 15: Half-profile of  $L\beta$  line at 1025Å

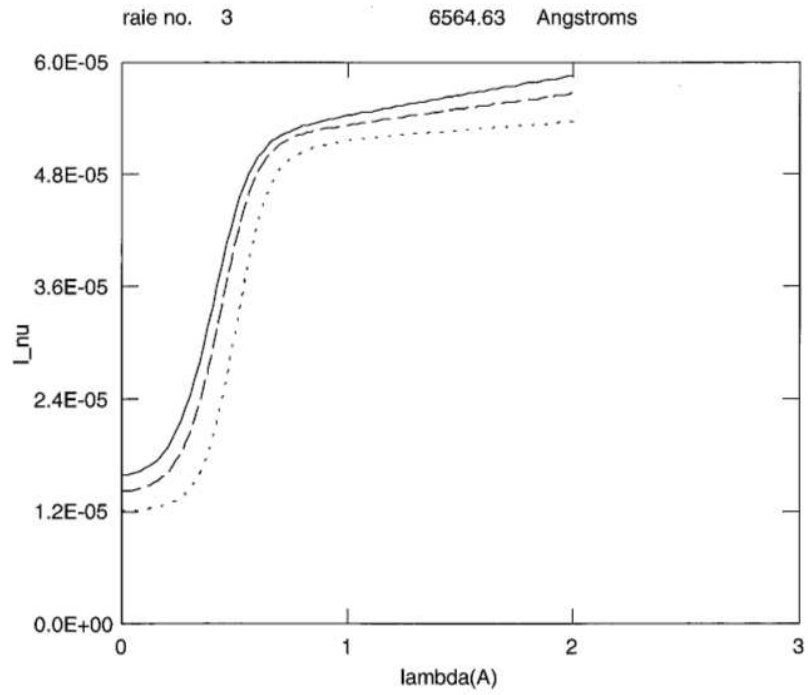


Figure 16: Half-profile of  $H\alpha$  line at  $6564\text{\AA}$

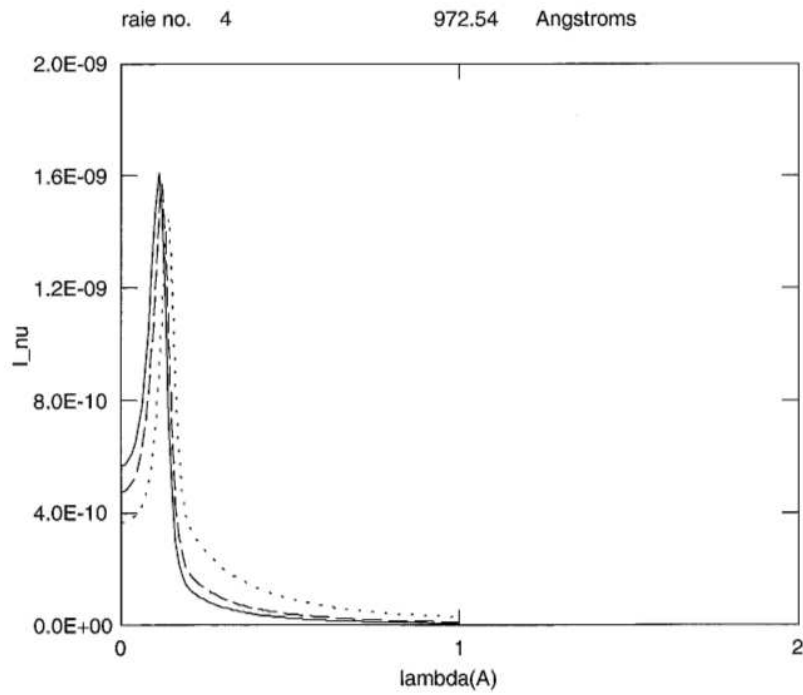


Figure 17: Half-profile of  $L\gamma$  line at  $972\text{\AA}$

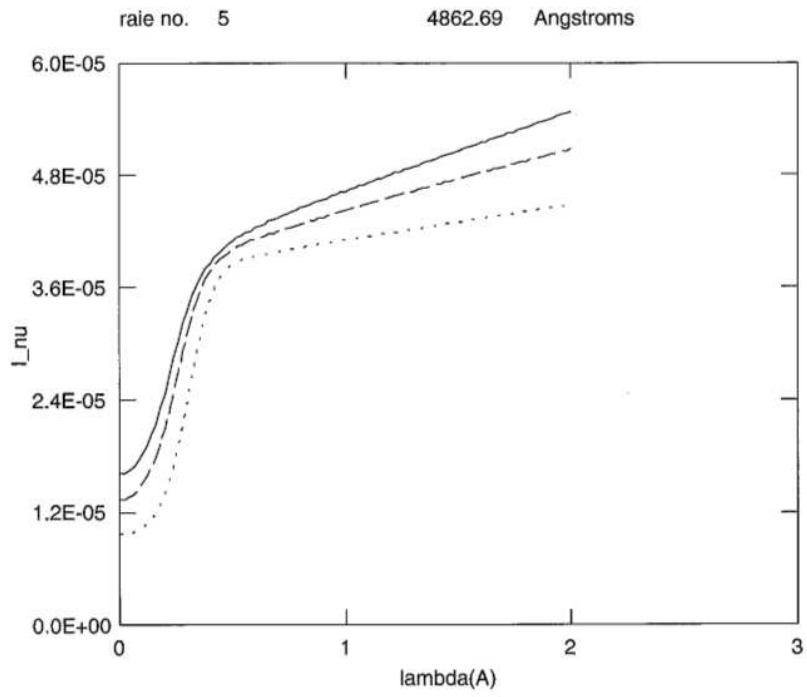


Figure 18: Half-profile of  $H\beta$  line at  $4862\text{\AA}$

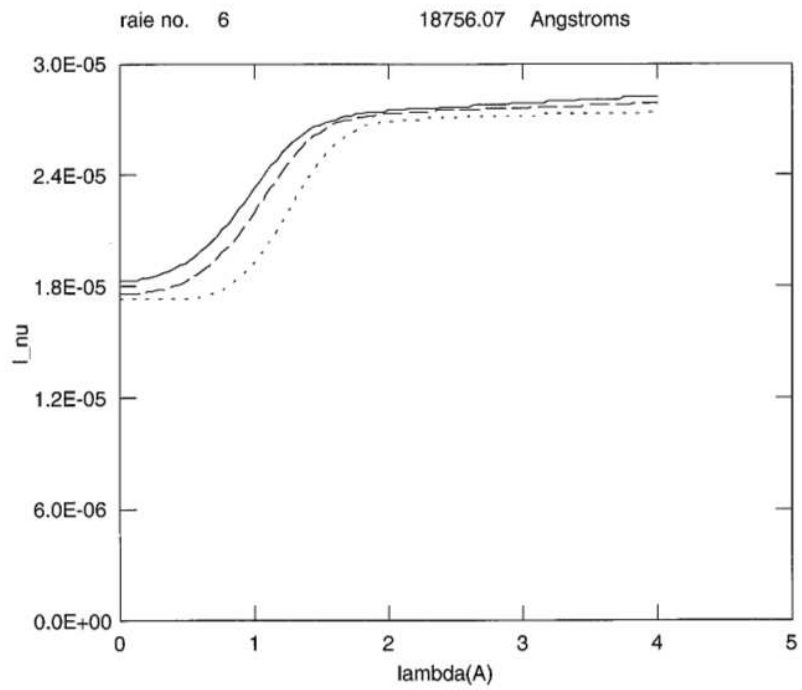


Figure 19: Half-profile of  $Pa - \alpha$  line at  $18756\text{\AA}$

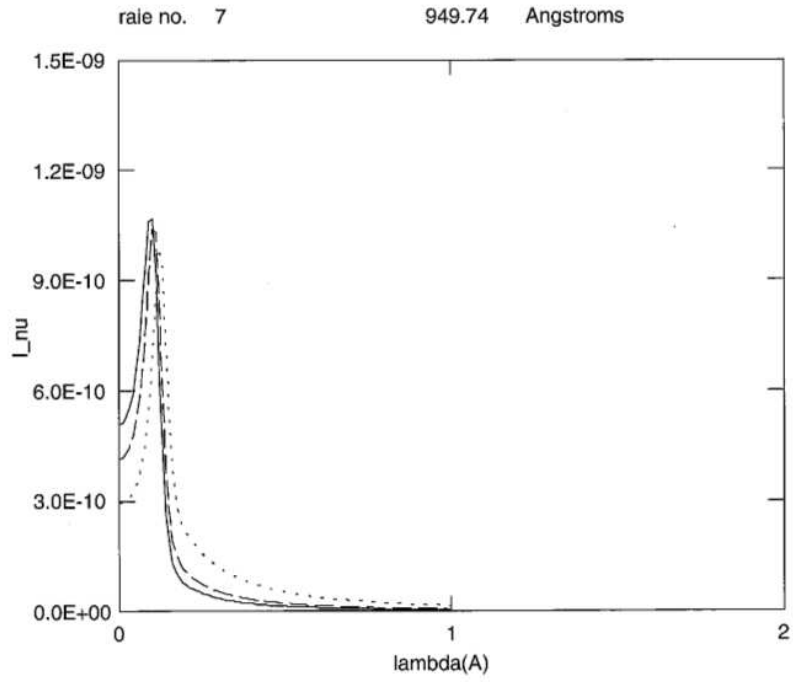


Figure 20: Half-profile of  $L\delta$  line at  $949\text{\AA}$

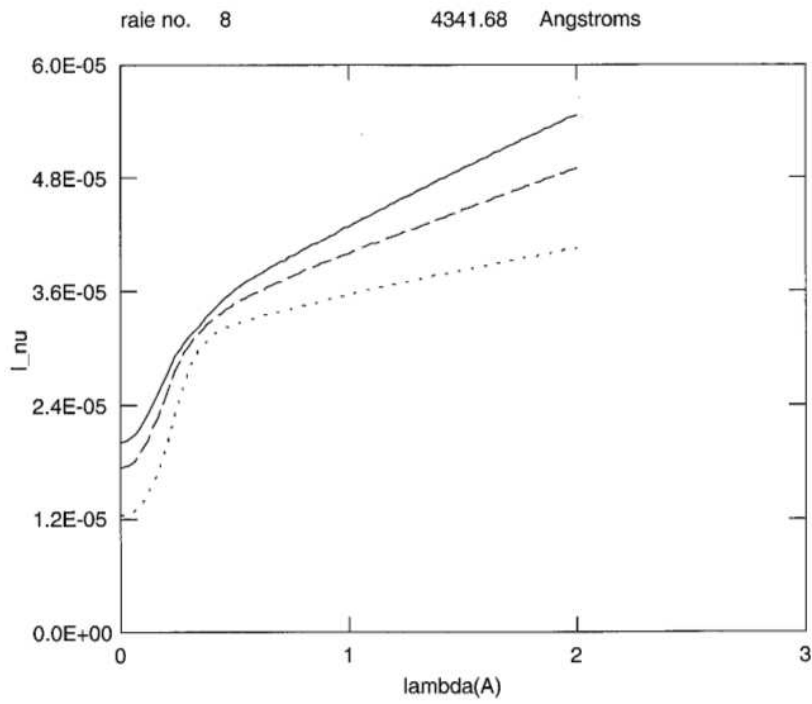


Figure 21: Half-profile of  $H\gamma$  line at  $4341\text{\AA}$

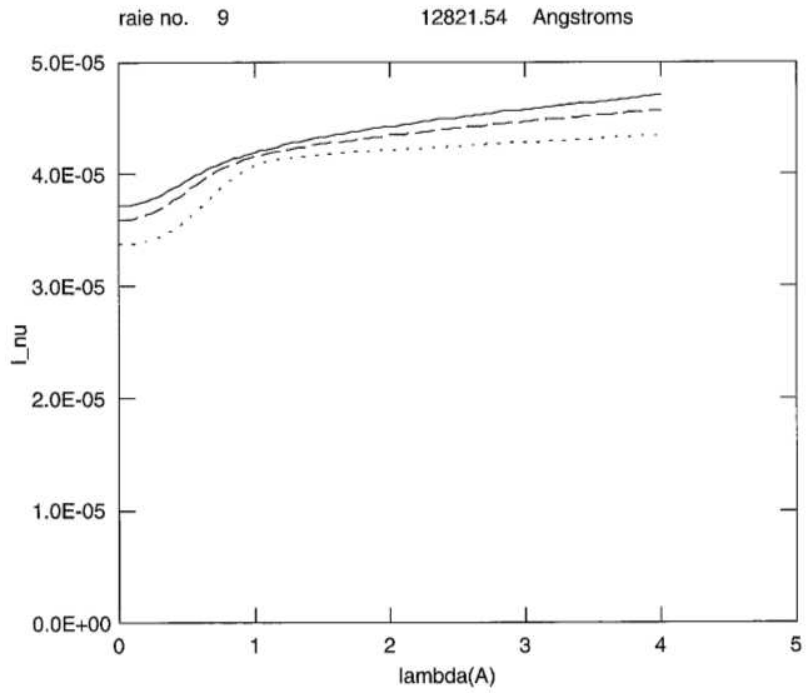


Figure 22: Half-profile of  $Pa - \beta$  line at  $12821\text{\AA}$

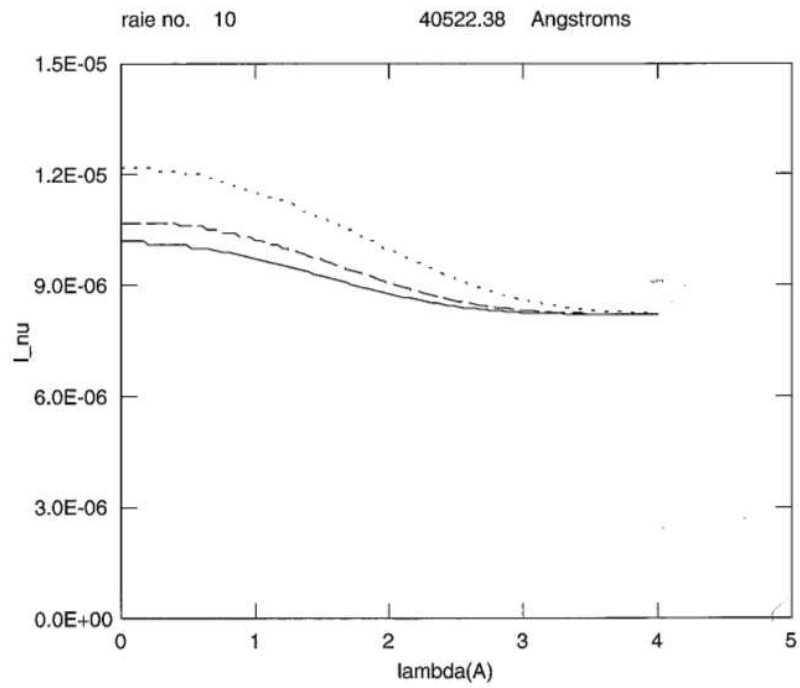


Figure 23: Half-profile of  $Br - \alpha$  line at  $40522\text{\AA}$

## 11 Running HYDR\_NV

- Download the package source file HYDR\_NV.tgz from MEDOC website:  
[https://idoc.ias.u-psud.fr/MEDOC/Radiative transfer codes](https://idoc.ias.u-psud.fr/MEDOC/Radiative%20transfer%20codes)
- gfortran compiler is required.
- Unpack the package by typing the following linux command:  
**tar -xvzf HYDR\_NV.tgz**
- Go to folder HYDR\_NV:  
**cd HYDR\_NV**
- The folder contains two cases of atmosphere: solar atmosphere (SUN folder corresponding to VAL3C model) and stellar atmosphere (ALTAIR folder). SUN and ALTAIR folders contain the following files: fort.3 (input), makefile, hydr\_nv.f90, visu21.f90
- **make**
- **./hydr\_nv**
- The output files are: fort.8 (iteration message), fort.21 (emergent intensities for 3 directions and for each transition line), fort.22 ( $L\alpha$  intensities versus frequency and emission angle), fort.24 (atmospheric parameters,  $N_e$ ,  $N_H$ ), fort.28 (mean intensities)
- The folder **results** contains the output files corresponding to a test case to be able to check if your results are good.
- To visualize line profiles (fort.21), use the following visualization program visu21.f90:  
**gfortran -o visu21 visu21.f90**  
**./visu21**  
The output file is **v21.ps**

## 12 CPU time

On a PC with 4 Intel processors (2.67 GHz):

- CPU time for ALTAIR (stellar) atmosphere model: 46.988 s < 1 min
- CPU time for a solar atmosphere model “VAL3C”: 28.094 s < 1 min



## 13 Acknowledgements

We thank MEDOC directors: Eric Buchlin (present) and Frédéric Baudin (former).

28th September 2015

M. C-Y

## References

- [1] C.W. Allen. *Astrophysical quantities*. The Athlone Press, University of London, 1973.
- [2] M. Arnaud and R. Rothenflug. An updated evaluation of recombination and ionization rates. *A&A suppl. Ser.*, 60:425–457, 1985.
- [3] L.H. Auer and D. Mihalas. On the use of variable Eddington factors in Non-LTE stellar atmospheres computations. *Mon. Not. R. astr. Soc.*, 149:65–74, 1970.
- [4] Eugene H. Avrett. *Introduction to Non-LTE radiative transfer and atmospheric modeling*. Lecture notes, 2008.
- [5] S. Chandrasekhar. *Radiative transfer*. Dover Publications, 1960.
- [6] CHIANTI Database. George Mason University (USA) and University of Michigan (USA) and University of Cambridge (UK). <http://www.chiantidatabase.org/>, Mai 2014.
- [7] K.P. Dere, E. Landi, H.E. Mason, B.C. Monsignori Fossi, and P.R. Young. CHIANTI - an atomic database for emission lines: I. Wavelengths greater than 50a. *A&A suppl. Ser.*, 125:149–173, 1997.
- [8] O. Gingerich. Review of Opacity Calculations. *Proceedings of the First Harvard-Smithsonian Conference on Stellar Atmospheres*, 167:1–25, 1964.
- [9] P. Gouttebroze and P. Heinzel. Calcium to hydrogen line ratios in solar prominences. *A&A*, 385:273–280, 2002.
- [10] P. Gouttebroze, P. Heinzel, and J.-C. Vial. The hydrogen spectrum of model prominences. *A&A Suppl. Ser.*, 99:513–543, 1993.
- [11] P. Gouttebroze and N. Labrosse. A ready-made code for the computation of prominence NLTE medels. *Solar Physics*, 196:349–355, 2000.
- [12] P. Gouttebroze, P. Lemaire, J.-C. Vial, and G. Artzner. The solar hydrogen lyman- $\beta$  and lyman- $\alpha$  lines : disk center observations from oso 8 compared with theoretical profiles. *APJ*, 225:655–664, 1978.
- [13] Pierre Gouttebroze. *Formation des raies spectrales et oscillations dans la chromosphère solaire*. Thèse d’Etat, Université Paris 7, 1980.
- [14] I. Hubeny and D. Mihalas. *Theory of Stellar Atmospheres : An introduction to Astrophysical Non-equilibrium Quantitative Spectroscopic Analysis*. Princeton University Press, 2015.
- [15] J.T. Jefferies. *Spectral line formation*. Blaisdell, 1968.

- [16] L.C. Johnson. Approximation for collisional and radiative transition rates in atomic hydrogen. *A&A*, 174:227–236, 1972.
- [17] Nicolas Labrosse. *Modélisation du spectre de l'hélium dans les protubérances solaires*. PhD thesis, Université Paris XI Orsay, 2001.
- [18] P. Lemaire, P. Gouttebroze, J.-C. Vial, and G. E. Artzner. Physical properties of the solar chromosphere deduced from optically thick lines: I. Observations, data reduction, and modelling of an average plage. *A&A*, 103:160–176, 1981.
- [19] J. Oxenius. *Kinetic Theory of Particles and Photons. Theoretical Foundations of Non-LTE Plasma Spectroscopy*. Springer-Verlag, 1986.
- [20] G. Peach. A general formula for the calculation of absorption cross sections for free-free transitions in the field of positive ions. *Royal Astronomical Society*, 130:361–377, 1964.
- [21] C. Pecker-Wimel. *Introduction à la spectroscopie des plasmas*. Gordon and Breach, 1966.
- [22] R.J. Rutten. *Radiative transfer in stellar atmospheres*. 2003.
- [23] E. Schatzman. *Astrophysique*. Masson et Cie, 1963.
- [24] J.E. Vernazza, E. H. Avrett, and R. Loeser. Structure of the Solar chromosphere; ii. The underlying photosphere and temperature-minimum region. *ApJS*, 30:1–60, 1976.