

USER GUIDE

PROM5 : Non-LTE radiative transfer in solar prominences

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Web access:

<https://idoc.osups.universite-paris-saclay.fr/medoc/tools/radiative-transfer-codes/>

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1 Introduction

The PROM5 code was written in Fortran 77 by P. Gouttebroze ([9], [10], [11]) and describes Non-LTE (NLTE) radiative transfer (1D) for hydrogen atom, in solar prominences, using partial frequency redistribution for resonance lines of hydrogen. We solve the equations of radiative transfer, statistical equilibrium of level populations, and ionization equilibrium for the hydrogen atom using an iterative method.

PROM5 is similar to PROM7 (Non-LTE radiative transfer in solar prominences and filaments for atmosphere models which are isothermal and isobaric) except that atmosphere models considered here vary inside the prominence. For a given model of atmosphere defined by temperature, T , pressure, P , microturbulent velocity, V , and thickness, e , the formation of hydrogen line is considered. Thus, we obtain the electron density and the emergent intensities for hydrogen lines. These lines (intensity, profile) are used to diagnose solar prominences through comparison with observations.

The original version of PROM5, adapted to **gfortran** compiler, is available from MEDOC website:

<https://idoc.osups.universite-paris-saclay.fr/medoc/tools/radiative-transfer-codes/>

In the following sections, we will explain in details PROM5 code (hydrogen): modeling, implemented equations, algorithm, numerical methods, etc.

2 Description of PROM5 code

Nature of the physical problem: NLTE radiative transfer (1D) in solar prominences (*HI* lines)

Method of solution: Feautrier method with variable Eddington factors

Restrictions on the complexity of the problem: PROM5 treats hydrogen atom in solar prominences

Other relevant information:

- The atmosphere model considered here varies inside the prominence
- Partial frequency redistribution (PRD) is used for resonance lines of hydrogen, while for the other lines and continua complete frequency redistribution (CRD) is used

Author: P. Gouttebroze

Program available from:

<https://idoc.osups.universite-paris-saclay.fr/medoc/tools/radiative-transfer-codes/>

Computer(s) on which program has been tested: PC with 4 Intel processors (2.67GHz)

Operating System(s) for which version of program has been tested: Linux

Programming language used: Adapted in Fortran 90 for **gfortran** compiler

Status: Stable

Accessibility: open (MEDOC)

No. of code lines in combined program and test deck: ~ 6000

Typical running time: < 1 min for one atmosphere model which varies inside the prominence

Reference:

- P. Gouttebroze, P. Heinzel and J.-C. Vial, “The hydrogen spectrum of model prominences”,
A&A Sppl. Ser., 99, 513-543, 1993

3 Algorithm

The figure 1 describes the algorithm of PROM5 code.

The algorithm starts with an initialization of physical parameters for hydrogen:

- Statistical equilibrium (see figure 4): A_{ji} (Einstein A coefficient), B_{ij} and B_{ji} (Einstein B coefficients), C_{ij} and C_{ji} (collisional excitation and deexcitation coefficients)
- Boundary conditions (incident intensities for H lines and continua)
- Radiation temperatures

The aim of PROM5 code is first to compute electron density. The free electrons are assumed to be obtained by ionization of hydrogen and the other elements (He, metals, etc), represented by the red loop in figure 1. Inside this loop, the statistical equilibrium is solved by iteration i.e. hydrogen level populations are calculated by taking $N_e = 1/2 N_H$ as a starting value (N_e and N_H are electron and hydrogen densities). By iteration, we obtain N_e and the atmosphere modeling for hydrogen, this one being the most abundant element in prominences. The hydrogen emergent spectrum is calculated for different values (here 3 values) of θ , angle between light ray and the normal to the solar surface.

Hydrogen (H I, II):

- Statistical equilibrium: A_{ji} , B_{ji} , B_{ij} , C_{ij} , C_{ji}
- Boundary conditions (incident intensities for lines and continua)
- Radiation temperatures

For an atmosphere model
which varies inside
the prominence

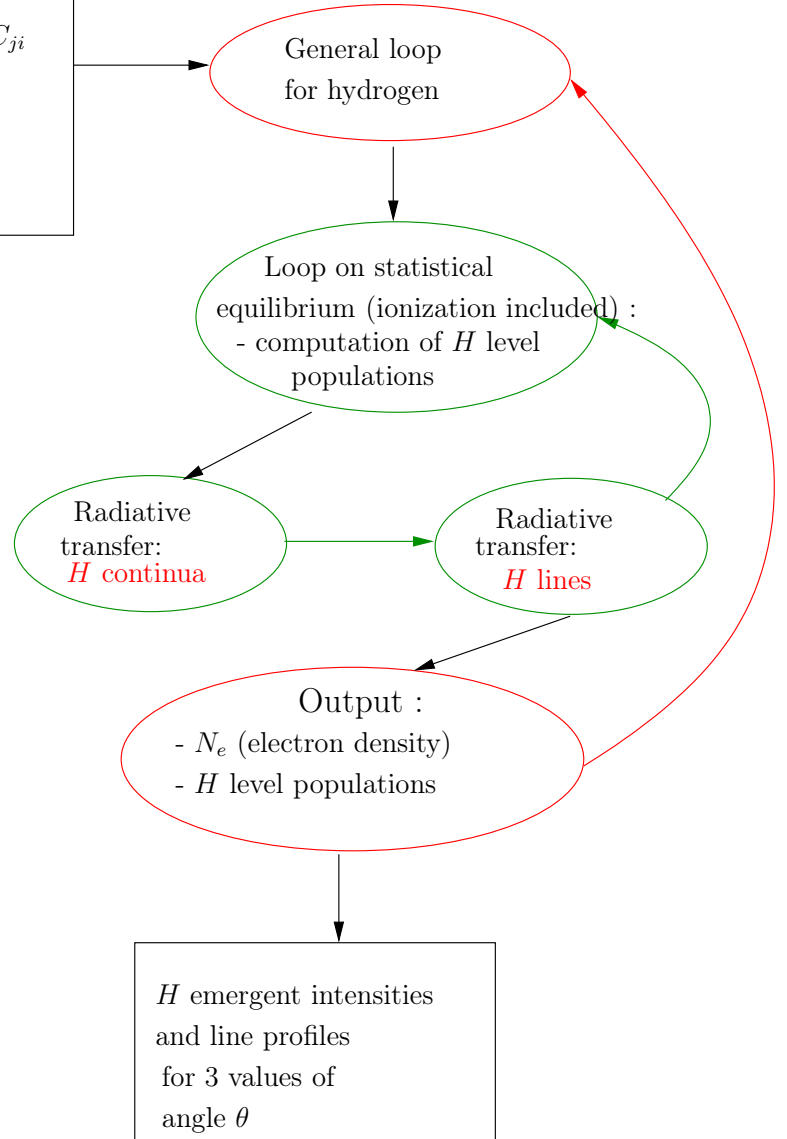


Figure 1: Algorithm of PROM5 code

4 Prominence modeling

Picture 2 represents an erupting prominence observed by SDO/AIA (2012/08/30) at 304Å and 171Å wavelengths.

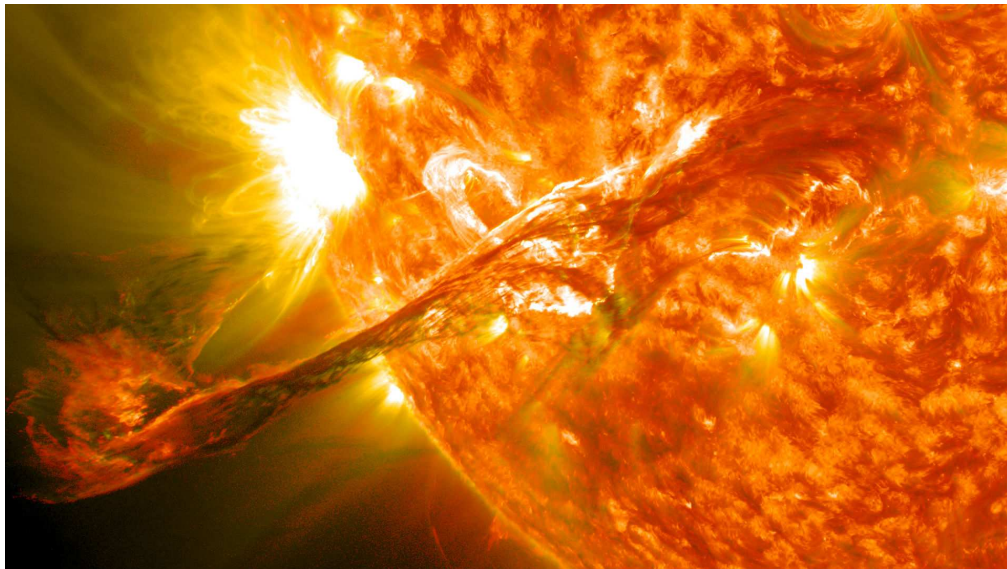


Figure 2: Erupting solar prominence observed by SDO/AIA, at 304Å and 171Å wavelengths, 2012/08/30

In the modeling (see figure 3), a prominence is represented by a symmetric plane-parallel slab (with thickness e) standing vertically above the solar surface (at altitude h) and irradiated on both faces by the Sun. It is a 1D representation. Inside the prominence, the initial condition is defined by 3 physical parameters varying with respect to the slab position: electronic temperature, T , pressure, P and microturbulent velocity, V . Due to the symmetry of the problem, calculations can be done in a half slab.

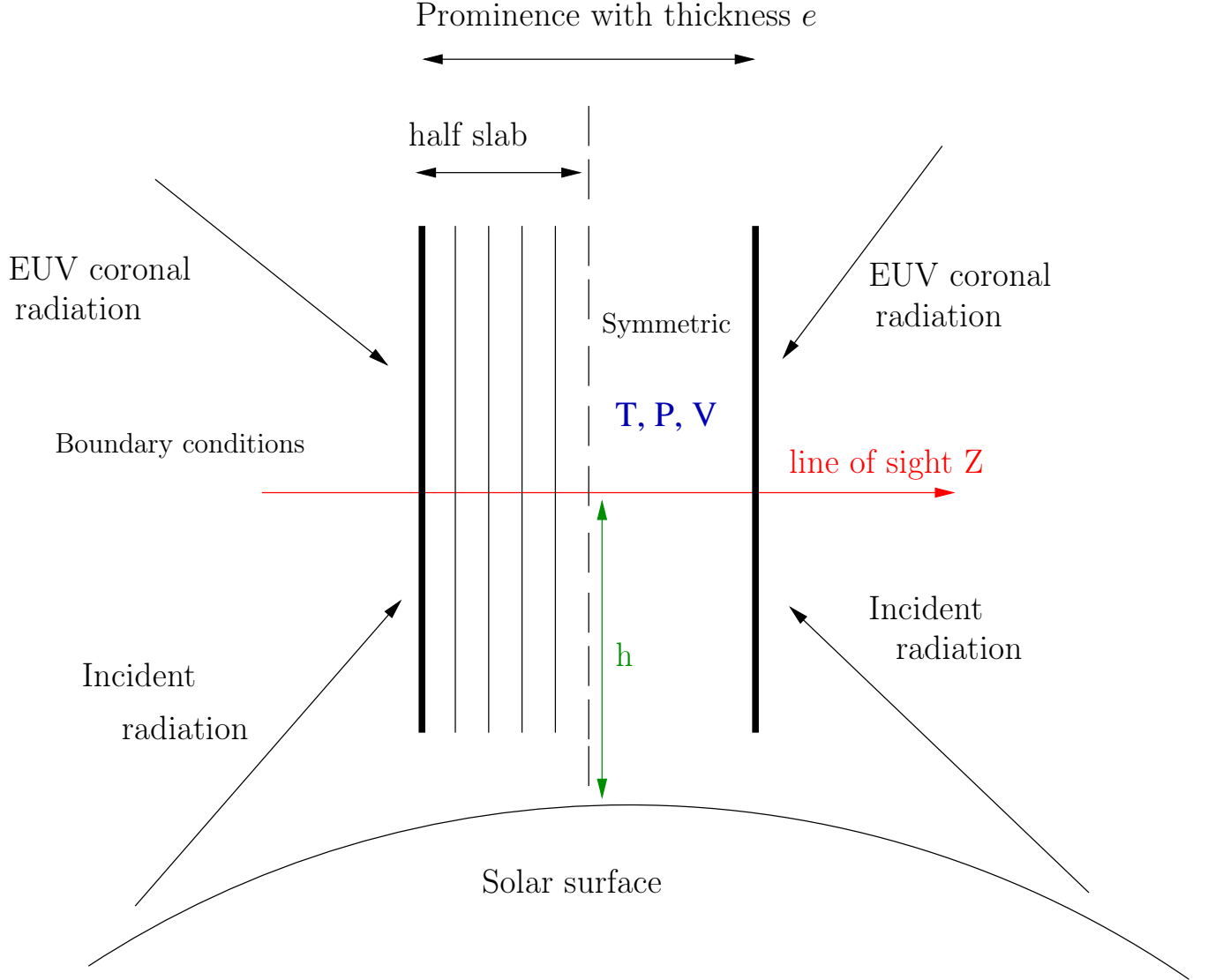


Figure 3: Prominence model

5 Bound-Free and Bound-Bound transitions

The transitions used in PROM5 code are Bound-Free and Bound-Bound transitions, which are represented in figure 4. Bound-Free transitions are between a bound state i and a continuum, producing a free electron with energy ϵ . It starts from excited states limit, i.e. $\epsilon = 0$. Bound-Bound transitions are from level i to level j .

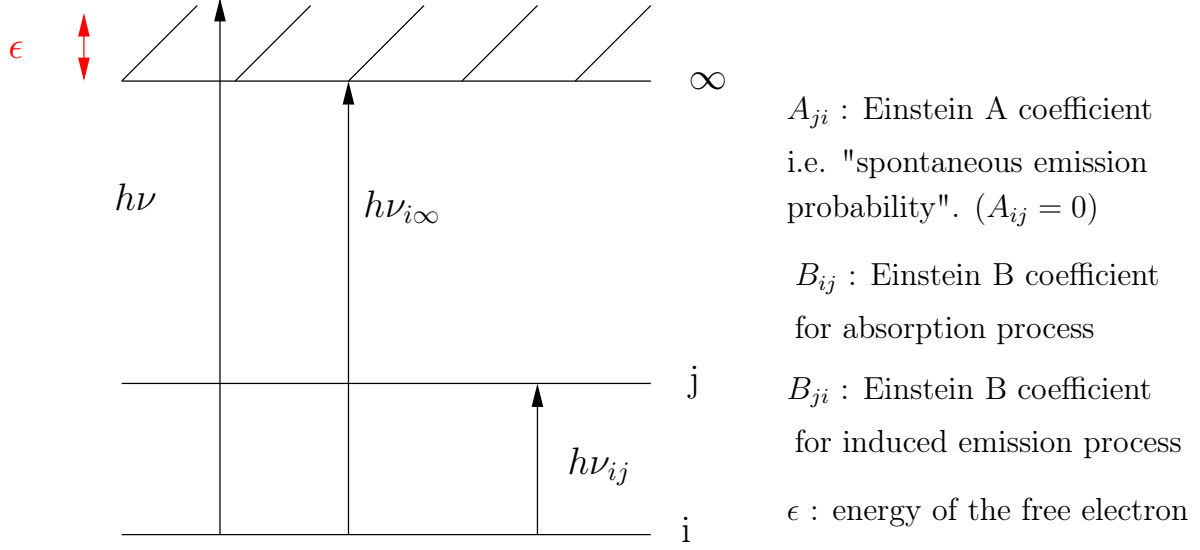


Figure 4: Transition types used in PROM5

6 Atomic structure of hydrogen

The transitions for hydrogen are summarized in Table 1. NN is the number of energy levels, including the threshold level continuum, bound levels limit (∞). $NTT = NN \times (NN - 1)/2$ is the total number of transitions, $NTAC = NN - 1$ is the number of continua (bound-free transitions) and $NTAR = NTT - NTAC$ is the number of lines (bound-bound transitions). In PROM5, $NN = 21$ (20 niveaux et 1 continu).

7 Population equations and radiative transfer equation

Let's consider level j in figure 4. At equilibrium, the population equations for this level contain all processes of population and depopulation of level j to other levels i which are balanced:

$$N_j \sum_{j \neq i}^n P_{ji} = \sum_{j \neq i}^n N_i P_{ij} \quad (7.1)$$

The depopulation rate P_{ji} from level j is written as :

$$P_{ji} = A_{ji} + B_{ji} \bar{J}_{ij} + C_{ji} = R_{ji} + C_{ji} \quad (7.2)$$

where C_{ji} is the collisional deexcitation rate, proportional to the electron density. R_{ji} is the radiative depopulation probability either by absorption and by spontaneous (A_{ji}) or by stimulated emission. A_{ji} and B_{ji} are corresponding to Einstein coefficients.

The population rate P_{ij} of the level j can also be written as :

$$P_{ij} = B_{ij} \bar{J}_{ij} + C_{ij} = R_{ij} + C_{ij},$$

Transition	Lower level	Upper level	Transition name	Wavelength $\lambda(\text{\AA})$
1	1	2	$Ly - \alpha$	1215
2	1	3	$Ly - \beta$	1025
3	2	3	$H - \alpha$	6564
4	1	4	$Ly - \gamma$	972
5	2	4	$H - \beta$	4862
6	3	4	$Pa - \alpha$	18756
7	1	5	$Ly - \delta$	949
8	2	5	$H - \gamma$	4341
9	3	5	$Pa - \beta$	12821
10	4	5	$Br - \alpha$	6563
11	1	6	$Ly - \epsilon$	973
12	2	6	$H - \delta$	4102
13	3	6	$Pa - \gamma$	10935
14	4	6	$Br - \beta$	4861
15	5	6	$Pf - \alpha$	7460
16	1	7	$Ly - 6$	931
17	2	7	$H - \epsilon$	3971
...
$NTAR + 1$	1	NN	Lyman continuum	911
$NTAR + 2$	2	NN	Balmer continuum	3645
...
$NTT =$	$NN - 1$	NN		

Table 1: Main transitions for hydrogen

where R_{ij} and C_{ij} are similar to R_{ji} and C_{ji} , except for spontaneous emission ($A_{ij} = 0$). The expression of \bar{J}_{ij} is:

$$\bar{J}_{ij} = \int_{-\infty}^{+\infty} J_{\nu} \Phi_{\nu} d\nu \quad (7.3)$$

where J_{ν} is mean intensity integrated over direction $\mu = \cos \theta$, θ being the angle between the light ray and the normal to the solar surface:

$$J_{\nu} = \frac{1}{2} \int_{-1}^1 I_{\nu}(\mu) d\mu \quad (7.4)$$

Φ_{ν} is the normalized profile of the absorption coefficient $\left(\int \Phi_{\nu} d\nu = 1 \right)$.

The left-hand side term in the population equations (7.1) represents all the processes which depopulate level j and the right-hand side term corresponds to all the levels which populate

the level j . And we use the following closure conservation equation:

$$\sum_{j=1}^n N_j = N_T \quad (7.5)$$

where N_T is the total population of the atomic element.

These population equations (7.1) which contain the mean intensities via \bar{J}_{ij} (i.e. integrated over the line profile centered in ν_0) are coupled with the radiative transfer equation (7.6). The intensity in the radiative transition is obtained by solving the transfer equation:

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu \quad (7.6)$$

τ_ν is the optical depth at frequency ν , and S_ν is the total source function defined by:

$$S_\nu = \frac{\epsilon_\nu}{\kappa_\nu},$$

where κ_ν is the absorption coefficient and ϵ_ν the emission coefficient.

Thus, we obtain a coupled equation system which is solved by numerical methods such as the Feautrier method ([12]). For more details, see the next two sections.

8 Formalism used for radiative transfer

The radiative transfer equations in the lines and continua of hydrogen are solved by a finite-difference method (Feautrier method) with variable Eddington factors. The other equations, namely that of statistical equilibrium of level populations, ionization and pressure equilibria, are solved by iteration. This is basically equivalent to a two-level atom model.

At the beginning, the intensities inside the slab are set by assuming that the medium is optically thin in all transitions (i.e. without radiative transfer). With these intensities, we compute the radiative transition rates. We start from an arbitrary electron density (half of the hydrogen density) to determine the collisional rates. Then, we solve the statistical equilibrium equations to determine hydrogen level populations and obtain new electron densities. The hydrogen densities are adjusted in order to satisfy the pressure equilibrium condition. After a few iterations, we obtain hydrogen and electron densities which are consistent with the intensities.

We compute the optical depths and solve the radiative transfer equations for the continua and the lines which are optically thick. The incident intensities are used as boundary conditions. The Lyman- α and β lines are treated according to standard partial redistribution, while complete redistribution is assumed for other lines. Since the slab is symmetrical, computations are performed in the half-slab only. These computations produce a new set of

intensities, which in turn yields new radiative rates, and new populations through the statistical equilibrium equations. In practice, “net radiative rates” (not explained here) in lines are used to control the convergence, with the help of relaxation parameters. This process is applied many times before new hydrogen and electron densities are computed to satisfy pressure and ionization equilibria.

The process stops when some fixed convergence criteria are satisfied both for net radiative rates in lines and radiation temperatures in continua.

9 Subroutine descriptions

PROM5 code starts by reading the input file “model5.dat”, by calling the main subroutine (P5EXE), by producing the output files such as line profiles given as data (“profil5.dat”) and by calling visualization subroutines (PSDEB, VISU21L, PSFIN). In section 9.2, each subroutine is explained in details.

Input files for PROM5:

- ★ intinc.dat : hydrogen incident intensities
- ★ model5.dat (for a given model defined in figures 5 and 6): file including the number of meshes ($NZ = 74$) to be computed, altitude (km) and 4 columns of data. The first column corresponds to temperature T (K), the second column to pressure P (dyn.cm^{-2}), the third column to prominence thickness (km) and the fourth column to microturbulent velocity (km.s^{-1}) along the line of sight (i.e. prominence thickness).

Output files for PROM5:

- ★ profil.dat: line profiles given as data
- ★ profil.ps: line profiles as PostScript file

In the next section, hydrogen modeling will be explained in details, such as all subroutines called by PROM5 code and a set of variables used in P5EXE subroutine.

9.1 Set of variables used in P5EXE subroutine

- IVERT=1: prominence model (vertical slab)
- NZ: number of meshes given by input file “model5.dat”
- NFR: number of frequencies in a line

- NMU: number of directions $\mu = \cos \theta$, where θ is the angle between the light ray and the normal to the solar surface
- NPSOR: number of points for the visualization of line profiles
- NFRC: number of frequencies in a continuum
- NTAB: size of the temperature array TAB (for the computation of collision rate coefficients)
- NINF: transition lower level
- NSUP: transition upper level
- ITP: to indicate if the transition is allowed or forbidden
- IOPRN = 1: complete redistribution (CRD), IOPRN = 3: partial redistribution (PRD), IOPRN = 4: PRD with coherent coefficient depending on frequency (particular case for $Ly\ \alpha$)
- IOPERA = 1: line profiles are printed, IOPERA = 0: line profiles are not printed. IOPERA is linked to DL2 (see below)
- DL1: x-axis (first wavelengths) for line profiles (visualization)
- DL2: y-axis (last wavelengths) for line profiles (visualization)
- IOPMRU = 1: only one redistribution matrix is computed for each transition (the same matrix is used for all meshes), IOPMRU = 0: all the redistribution matrices are computed for each transition (the same matrix is used for all meshes)
- EPS and BEN: line coupling coefficients
- CSI0 and ETA0: parameters for continua which are equivalent to EPS and EPSxBEN for lines
- GIBAR: mean intensity weighted by absorption profile
- IOPECO: equivalent to IOPERA for continua
- IOPFEV: option for computing variable Eddington factors
- IPROF = 1: optical depths are printed. If not, IPROF = 0
- IOPMIC = 1: microturbulent pressure is taken account. If not, IOPMIC = 0
- NVLI: number of lines to visualize
- IVHYD: visualization option for hydrogen

- INIDLD: option for initializing automatically Doppler widths from temperature of reference TERE (for graphic representation)
- IOPCAC = 0: without continuum absorption, IOPCAC = 3: continuum absorption is included in computations
- IOPEAC: prints continuum absorption
- IOPATM: prints atmosphere parameters
- IOP TTC: prints continuum transition rates
- PMU: values of μ (line profiles for $\mu = 1$ are plotted in plain line, for $\mu = 0.6$ in dashed line, and for $\mu = 0.2$ in dotted line)
- IVTR: visualization index for radiative transfer
- ETOT: total energy emitted by prominence for a line
- RDMAT: redistribution matrix
- AJI: Einstein A coefficient i.e. spontaneous emission
- BJI, BIJ: Einstein B coefficients for absorption and induced emission processes
- CIJ, CJI: collisional excitation/deexcitation coefficients
- CAC: continuum absorption coefficient
- TR: radiation temperature for bound-free transition
- FADIR: dilution factor for lines
- FADIC: dilution factor for continua
- CPOL: polynomial coefficients of degree 5 used in the computations of dilution factors (array size : NCMX)
- FEVK and FEVL, FKTC and FLTC : variable Eddington factors (resp. lines and continua)
- HIEMC: intensity emitted in the continuum
- RIK, RKI: radiative transition rates
- FIIR, FISR: lower and upper incident line flux
- FRR: frequency in the line

- Atmosphere parameters (array size NZ) : XM (column-mass), Z (position in the slab), TE (temperature), PG (gas pressure), VT (microturbulent velocity), HNH (hydrogen density: number of atoms per unit volume), HNE (electron density)
- NTR et NTC: discrete and bound-free transition numbers
- ICTR and ICTC: control index for lines and continua
- FRN: level frequency
- XFR and XFRC: division model in frequency for lines and continua
- BRN and BRP: net radiative bracket before and after computations (they are used to control the convergence)
- SR and SC: line and continuum source functions
- STR: total source function (for line and continuum)
- CARR: line absorption coefficient
- COHER: coherent coefficient
- OIS: collisional ionization rate coefficient

9.2 Hydrogen modeling (Johnson model, [14])

Hydrogene modeling (main subroutine: P5EXE) is divided in several parts:

- INIMET: calculates metal abundances relative to hydrogen
- Input: atmosphere model parameters which vary inside the prominence (“model5.dat”)
- INIFEVH: initialization of Eddington factors (lines and continua)
- INIHV3 : initialization of atomic parameters for statistical equilibrium (AJI : Einstein A coefficient, BIJ and BJI : Einstein B coefficients, S_i et S_e : ionization and excitation coefficients)
- SOLINH: incident intensities (input file “intinc5.dat”)
- INTALT: computation of dilution factors (lines and continua) depending on altitude ($FADIR$: dilution factors for lines, $FADIC$: dilution factors for continua)
- COLHV3: boundary conditions for hydrogen (intensities)
- Computation of the integral over the line profile $HJBAR = \int_{-\infty}^{+\infty} J_{\nu} \Phi_{\nu} d\nu$

- INITR1: initialization of radiation temperatures for hydrogen (corresponding to the radiation of a black body) inside the slab from intensities computed before
- General loop for hydrogen:
 - do ITG = 1, ITGMAX*
 - HYESV3: computation of hydrogen level populations and electron density
 - *do IT = 1, NTAR*: loop on discrete transitions
 - RPCDEH: computation of partial and complete redistribution matrices
 - enddo*
 - If $IOPFEV \geq 3$, Eddington factors for lines and continua are printed via ECRFEV subroutine
 - Parameters storage for lines (*HJBAR* et *BRN* (net radiative bracket)) and continua (radiative excitation/ionization probabilities *RIK* and *RKI*) and for hydrogen level populations *HN*
 - *do icr = 1, ICRMAX*: loop on pressure equilibrium
 - Ionization of hydrogen determines free electron density.
 - * EQSTHV3: Solution of statistical equilibrium equations for hydrogen excited level populations (*HN*). More precisely, statistical equilibrium is computed again because some transition coefficients P_{ji} defined by (7.2) changed (for lines and continua)
 - * Bound-Free transitions:
 - ★ *do itac = 1, NTAC*: loop on continua
 - . HCONTI: NLTE radiative transfer for continua. Intensities and transition probabilities P_{ji} defined by (7.2) are computed
 - ★ *enddo* (for *itac*)
 - ★ COMPACH: comparison for convergence
 - * Bound-Bound transitions:
 - ★ *do itar = 1, NTAR*: loop on lines
 - . RPRHV3: NLTE radiative transfer for lines
 - ★ *enddo* (for *itar*)
 - ★ COMPARCH: comparison for convergence
 - ★ Preparation of the next iteration: radiative transition coefficients *RIK* are computed from radiation temperature *TR* calculated before,
 - NOVRIK: for continua
 - NOVBRV3: for lines.
 - *enddo* (for *icr*)
 - NOVTR2: computation of new radiation temperature for bound-free transitions
 - Convergence and parameters storage for *HJBAR*, *HN*, *BRN*, *RIK*, *RKI*

enddo (for *ITG*)

- Prints Eddington factors *FEVK* (lines) and *FKTC* (continua)
- Output results: quantities are computed once again in order to obtain convergence results at the last iteration. Line profiles from results obtained at the last iteration are computed.
 - * EXOATM: prints atmosphere parameters (pressure, altitude, electron temperature, hydrogen density, etc)
 - * ABSCON: continuum absorption due to different elements is computed (defined in INIMET subroutine)
 - * EQSTHV3
 - * For lines
 - * RPCDEH
 - * RPRHV3
 - * SUMRA: prints wavelengths *AMBDA*, etc
 - * PROFLI: line profiles
 - * For continua
 - * HCONTI
 - * COMPARH
 - * COMPACH

Subroutines called above are explained in the next paragraph.

INIMET

INIMET computes total weight C_1 of elements and total abundance C_2 with respect to hydrogen of the $NEL = 20$ following elements: He, C, N, O, Ne, Na, Mg, Al, Si, P, S, Ar, K, Ca, Cr, Mn, Fe, CO, Ni, H.

Outputs : C_1 et C_2

INIFEVH

Initialization of line (*FEVK* and *FEVL*) and continua (*FKTC* et *FLTC*) Eddington factors at each frequency and for each mesh.

Inputs : *NZ* (number of meshes), *NTAR* (number of lines), *NTAC* (number of continua), *NFR* (number of frequencies in a line), *NFRC* (number of frequencies in a continuum)

Outputs : *FEVK*, *FEVL*, *FKTC* et *FLTC*

INIHV3

Computation of hydrogen atomic parameters for statistical equilibrium: Einstein A and B coefficients for photons (AJI , BIJ , BJI : independent of electron temperature), collisional excitation and ionization rate coefficients (OIS in PROM5 code) for electrons, called S_e and S_i in the formulas below. These coefficients are independent of electron temperature.

The model used for hydrogen is L.C. Johnson model ([14]).

AJI is obtained by AEMS function (in PROM5 code) which is calculated from “Gaunt factors” (Table 1 in [14]).

BJI is obtained from AJI , BIJ is such that $\omega_i B_{ij} = \omega_j B_{ji}$, where ω_i is the statistical weight associated to level i .

Computation of collisional excitation rate coefficients (CECH function in PROM5 code):

Let n and n' be two levels ($n < n'$). The excitation rate coefficient is given by the following formula (Johnson, [14]), implemented in PROM5 code:

$$S_e(n, n') = (8kT/\pi m)^{1/2} \frac{2n^2}{x} \pi a_0^2 y^2 \left(A_{nn'} \left[\left(\frac{1}{y} + \frac{1}{2} \right) E_1(y) - \left(\frac{1}{z} + \frac{1}{2} \right) E_1(z) \right] \right) \\ + (8kT/\pi m)^{1/2} \frac{2n^2}{x} \pi a_0^2 y^2 \left(\left[B_{nn'} - A_{nn'} \ln \frac{2n^2}{x} \right] \left[\frac{1}{y} E_2(y) - \frac{1}{z} E_2(z) \right] \right) \quad (9.1)$$

$E_i(z) = \int_1^\infty e^{-zt} t^{-i} dt$ is called Exponential integral of order i ($i = 0, 1, 2, \dots$).

m denotes the electron mass and $a_0 = 0.5292 \cdot 10^{-8} \text{ cm}$ the Bohr radius.

Here and below, E_n denotes level energy of n for hydrogen. We obtain:

$$y = (E_{n'} - E_n)/kT, \\ z = r_{nn'} + y, \\ x = 1 - (n/n')^2, \\ B_{nn'} = \frac{4n^4}{n'^3} x^{-2} \left(1 + \frac{4}{3} x^{-1} + b_n x^{-2} \right), \\ b_n = n^{-1} (4 - 18.63n^{-1} + 36.24n^{-2} - 28.09n^{-3}), \quad n \geq 2, \\ A_{nn'} = 2n^2 x^{-1} f_{nn'}, \\ f_{nn'} = \frac{32}{3\sqrt{3}} \frac{n}{\pi n'^3} x^{-3} g(n, x), \\ g(n, x) = g_0(n) + g_1(n) x^{-1} + g_2(n) x^{-2}, \\ r_{nn'} = r_n x, \\ r_n = 1.94n^{-1.57}$$

where g_0 , g_1 , g_2 : Gaunt factors for pour Bound-Free transitions. They are given in Table 2

(Table 1 in [14]) :

	$n = 1$	$n = 2$	$n \geq 3$
$g_0(n)$	1.11330	1.0785	$0.9935 + 0.2328n^{-1} - 0.1296n^{-2}$
$g_1(n)$	-0.4059	-0.2319	$-n^{-1}(0.6282 - 0.5598n^{-1} + 0.5299n^{-2})$
$g_2(n)$	0.07014	0.02947	$n^{-2}(0.3887 - 1.181n^{-1} + 1.470n^{-2})$

Table 2: Gaunt factors

Computation of collisional ionization rate coefficients (CICH function in PROM5 code):

Let us consider here level n . Ionization rate coefficient is given by the following formula (Johnson, [14]) implemented in PROM5 code:

$$\begin{aligned}
 S_i(n) = & (8kT/\pi m)^{1/2} 2n^2 \pi a_0^2 y_n^2 A_n \left[\frac{1}{y_n} E_1(y) - \frac{1}{z_n} E_1(z_n) \right] \\
 & + (8kT/\pi m)^{1/2} 2n^2 \pi a_0^2 y_n^2 (B_n - A_n \ln 2n^2) [\zeta(y_n) - \zeta(z_n)],
 \end{aligned} \tag{9.2}$$

where m is the electron mass and

$$\begin{aligned}
 y_n &= I_n/kT, \\
 z_n &= r_n + I_n/kT, \\
 \zeta(t) &= E_0(t) - 2E_1(t) + E_2(t)
 \end{aligned}$$

where I_n is ionization energy of level n for hydrogen.

Inputs: NZ , NN , NTT , $NTAR$, $NTAC$, $NFRC$, $NTAB$, $INIDLD$, $ICTR$, $VTUR$, $TSLAB$, NTR , NTC , TAB , $IVISU$

Outputs: AJI , BIJ , BJI , OIS , $DLDREF$ in frequency and $DNDREF$ in wavelength (Doppler width for Bound-Free transitions). Initialization of BRP , BRN , $FPES$, CAC , SC , $DCAC$

SOLINH

Definition of incident intensities for hydrogen lines in order to provide boundary conditions for the solution of radiative transfer equations.

The input file “intinc.dat” contains incident intensities for $NLI = 17$ hydrogen lines. The input file “tembri.dat”, which is a table of brightness temperatures, is used to compute intensities $FINT$ for the other hydrogen lines which are not in “intinc.dat”.

Inputs: *NN*, *NTT*, *NTAR*, *NFIMX*, *NCMX*, *NINF*, *NSUP*, *NFR*, *NTR*, *ICTR*, *NFINT*

Outputs: *FINT*, *CPOL*, *FRFI*

INTALT

Computation of dilution factors taking into account the limb darkening and prominence altitude. There is no obvious limb darkening for Lyman lines.

Inputs: *ALTI*, *NTAR*, *NTAC*, *NCMX*, *CPOL*

Outputs: *FADIR* (for lines) and *FADIC* (for continua).

COLHV3

Boundary conditions (hydrogen) for Bound-Free (continua) and Bound-Bound (lines) transitions.

For lines: lower boundary condition *FIIR* is null. *FLJ* is the solar flux (intensity) and *FADIR* is the dilution factor (see **INTALT** subroutine). Upper boundary condition *FISR* is equal to $FADIR \times FLJ$.

Moreover, natural broadening is computed (associated to level lifetime) for permitted lines by adding the *AJI* concerned (inverse of level lifetime). This broadening enables the computation of the profile Φ_ν . Profile line is renormalized (intensity): $GIOM = \int J_\nu \phi_\nu d\nu$.

For continua: lower boundary condition *FIIC* is null. *FADIC* is the dilution factor dilution (see **INTALT** subroutine) and B_ν is the Planck function. Upper boundary condition *FISC* is equal to $B_\nu \times FADIC$.

Inputs: *IVERT*, *NZ*, *NN*, *NTT*, *NTRD*, *NTRC*, *NFR*, *NFRC*, *NFIMX*, *ITRD*, *ICTR*, *NINF*, *NSUP*, *FRN*, *DNDREF*, *XFR*

Output: *GIOM*

INITR1

Initialization of radiation temperature (*TR*) corresponding to photoionization rates in hydrogen Bound-Free transitions, from incident radiation temperature and dilution factor *FADIC*. We assume at the beginning that the medium is optically thin in all transitions. The photoionization cross-section *SEP* for hydrogen is obtained from Gaunt factors ([8]). Radiation temperature changes during the iterative process.

Inputs: NZ , NN , NTT , $NTAC$, $NFRC$, NTC , $NINF$, FRN , $XFRC$, $FADIC$, RIK , TE

Output: TR

HYESV3

Solution of statistical equilibrium and ionization for given temperatures and pressures: hydrogen level populations and electron density are computed.

The 4 following subroutines are called in HYESV3:

TXCOLL: collisional transition probabilities (CIJ et CJI) are computed by using electron density.

INIRIKH: photoionization probabilities RIK (photoionization) and RKI (radiative recombination) are computed. Electron density is used for recombination but not for photoionization.

EQSTHV3: solution of statistical equations for hydrogen level populations by iteration. We obtain a linear system to solve. Output : HN for each level and mesh.

SAHARA: computation of electron density HNE and density of the other elements $HNION$. Saha law at LTE gives the ionization degree of each ion. We use here HNH (and not HN).

Inputs for HYESV3: NZ , NN , NTT , $NTAR$, $NTAC$, $NFRC$, $C1$, $C2$, ITP , $NINF$, $NSUP$, NTC , FRN , POM , $XFRC$, Z , XM , TE , VT , HNH , HNE , HN , $NTAB$, TAB

Outputs for HYESV3: CIJ , CJI , RIK , RKI , HNE , $HNION$

ABSCON

ABSCON is called when $IOPCAC = 3$ (radiative transfer doesn't take into account this effect). ABSCON computes continuum absorption in the neighbourhood of lines under consideration and uses PROFSC subroutine.

PROFSC : computes additional optical depth $TAUC$ (due to continuum absorption process), continuous source functions FSC , coherent scattering ratios $CSCAT$ for a given frequency $FREQ$, and continuous absorption coefficient $CABCO$.

To compute $CABCO$, we need continuum absorption (LTE) by hydrogen negative ions, aluminium, hydrogen (photoionization cross-section $SEPION$ is computed from Gaunt factors ([8])), carbon, magnesium, silicon and iron.

Inputs: $IVISU$, NZ , NN , NTT , $NTRD$, $NNHYD$, TE , Z , HNH , HNE , $XNHYD$, $ITRD$, $NINF$, $NSUP$, FRN , SC , $DCAC$, CAC

Outputs for ABSCON: $TAUC$, FSC , $CSCAT$, $CABCO$

RPCDEH (for resonance lines)

Hydrogen partial redistribution. Broadening constants for lines (natural and collisional) are computed.

RPCDEH uses the following subroutines:

ELCOH1 : computation of collisional broadening *DFRCO* for hydrogen lines

COHEVA : computation of coherent coefficient for different frequencies of *Ly α* line (H)

MARALA : computation of the redistribution matrix for *Ly α* line

MAREDI : computation of the redistribution matrix for the other lines (case of a resonance line with a frequency-independent coherence coefficient)

Inputs for RPCDEH: *IRS, IOPMRU, NZ, NN, NTT, NTRD, NFR, IVISU, IOP, ITRD, NINF, NSUP, FRN, AE, TE, VT, HNE, HN, DND, AM, COHE, FR, PHIX*

Outputs for RPCDEH: RDMAT (redistribution matrix)

ECRFEV

Prints variable Eddington factors *FEVK*

HCONTI

Solution of NLTE radiative transfert equations for hydrogen Bound-Free transitions in order to compute intensities and transition probabilities (P_{ji} is proportional to intensities and is used for statistical equilibrium).

HCONTI uses the following subroutines:

CSIETA: computation of *CSI0* and *ETA0* which are for continua, the equivalents of ϵ and $\epsilon \times B$ for lines (coupling coefficients)

PROFSC: see ABSCON subroutine

TFCFEV: solution of radiative transfer equations for continua, with external absorption sources (outputs: intensity *GI* inside the slab and mean intensity *GIBAR*) and source functions *S*. We use Feautrier method and variable Eddington factors

NOVFEV: computes new Eddington factors (outputs : *FEVK* et *FEVL*) from source functions ([3]). This subroutine concerns only discrete and Bound-Free transitions

TFCOM: is equivalent to TFCFEV subroutine in the case where the medium is optically thin (outputs : *GI, GIBAR* and *S*)

TRALA: computes emergent intensity *HIEMC* (integral of the source function) by using transition probabilities

Inputs for HCONTI: *IT, IVERT, NZ, NN, NTT, NTAR, NTAC, NFRC, NMU, NMUV, IOPECO, IOPFEV, ITP, NTC, NINF, FRN, FRC, POM, TE, HNH, HNE, Z*

Outputs for HCONTI: *HIEMC* (emergent intensity) and *RIKP* (transition probabilities for continua)

COMPACH

Comparison for convergence : *RIK* (hydrogen radiative transition parameter at previous iteration) is compared with *RIKP* (parameter at final iteration)

RPRHV3

Radiative transfer is computed for hydrogen lines.

RPRHV3 uses the following subroutines:

EPSBHV3 (preparation for radiative transfer): computation of coupling coefficients *BEN* for a hydrogen two-level atom model (output : *BEN*). *BEN* is used to transform intensity into source function which is useful to calculate emergent intensity.

TFRFEV : solution of NLTE radiative transfer equations for lines using Feautrier method (finite-difference method) and variable Eddington factors. The intensities inside meshes and source functions are computed (outputs : main intensity *GIBAR* and intensity inside the slab *GI*. We deduce source function *SR* in the line and the total source function *STR*)

NOVFEV : computes the new Eddington factors *FEVK* and *FEVL* (see HCONTI subroutine)

Inputs for RPRHCF : *IRS*, *NZ*, *NN*, *NTT*, *NTRD*, *NTRC*, *NFR*, *NMU*, *IVERT*, *Z*, *TE*, *NTR*, *NINF*, *NSUP*, *AJI*, *BIJ*, *BJI*, *HJBAR*

Outputs for RPRHCF : *BEN* (coupling coefficient for lines), *SR* and *STR* (source functions), *GI* and *GIBAR* (intensity inside the slab and mean intensity), *BRN1* (Net Radiative Bracket)

COMPARH

Comparison for convergence : *BRN* (at previous iteration) is compared with *BRP* (at final iteration). *BRN* is linked to statistical equilibrium and is calculated as a fonction of intensities.

NOVRIK

Computation of radiative transition coefficients for continua.

Input: *NZ*, *NTAC*, *ICTC*, *RXIK*, *RIKP*

Output: *RIK*

NOVBRV3

Computation of radiative transition coefficients for lines.

Inputs : NZ , $NTRD$, $ICTR$, $RXRO$

Output : BRN

NOVTR2

Update of radiation temperature (TR) for hydrogen Bound-Free transitions.

Inputs: ITG , $IVTR$, $RXTR$, NZ , NN , NTT , $NTAC$, $NFRC$, $ICTC$, NTC , $NINF$, FRN , $XFRC$, $RIKP$, TE

Output: TR

PROFLI

Computation of emergent line profiles for different values of $\mu = \cos \theta$.

Inputs: IRS , NZ , NN , NTT , $NTRD$, NFR , $NMUV$, $NPSOR$, $ITRD$, $NINF$, $NSUP$, FRN , $DL1$, $DL2$, PMU , FR , STR , DND , AM , CAR , CAC , TO , $NFINT$, $FRFI$, $FINT$, $TOTO$, $ETOT$, $FWHM$

Outputs: XX (wavelength range), YY (emergent intensity)

10 Results for hydrogen lines in prominences

The half-profiles are given for 3 values of $\mu = \cos \theta$ for a given model (see figures 5 et 6, $V_T = 5 \text{ km.s}^{-1}$).

The half-profile for $\mu = 1$ is plotted by a plain line. The half-profile for $\mu = 0.6$ is plotted by a dashed line and the half-profile for $\mu = 0.2$ by dotted line.

The following figures for hydrogen lines are stored in the folder **results** when you download the package source file PROM5.tgz from MEDOC website.

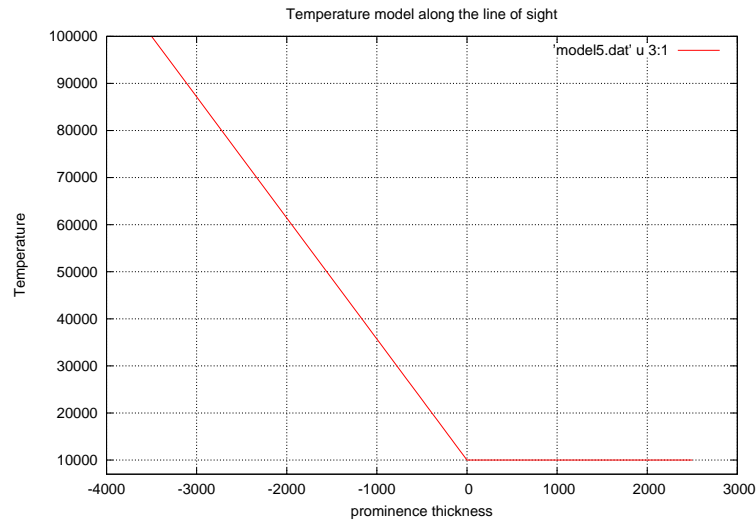


Figure 5: Temperature with respect to prominence thickness

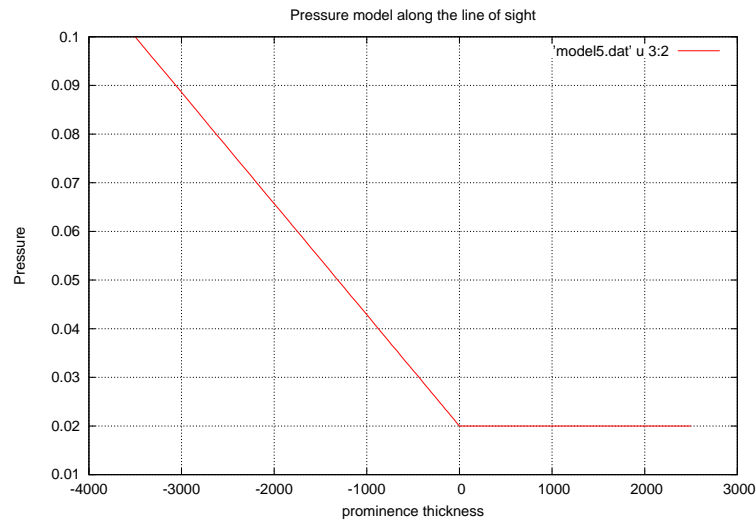


Figure 6: Pressure with respect to prominence thickness

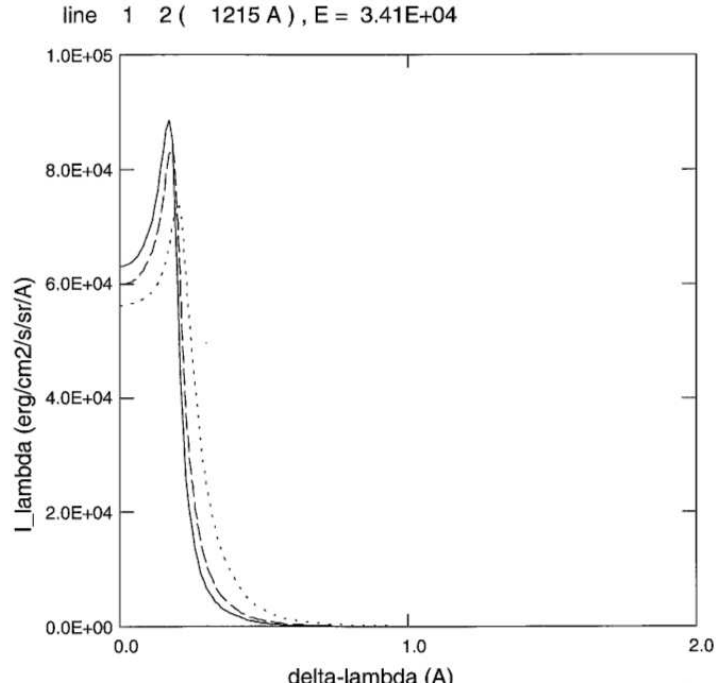


Figure 7: Half-profile of $L\alpha$ line at 1215Å

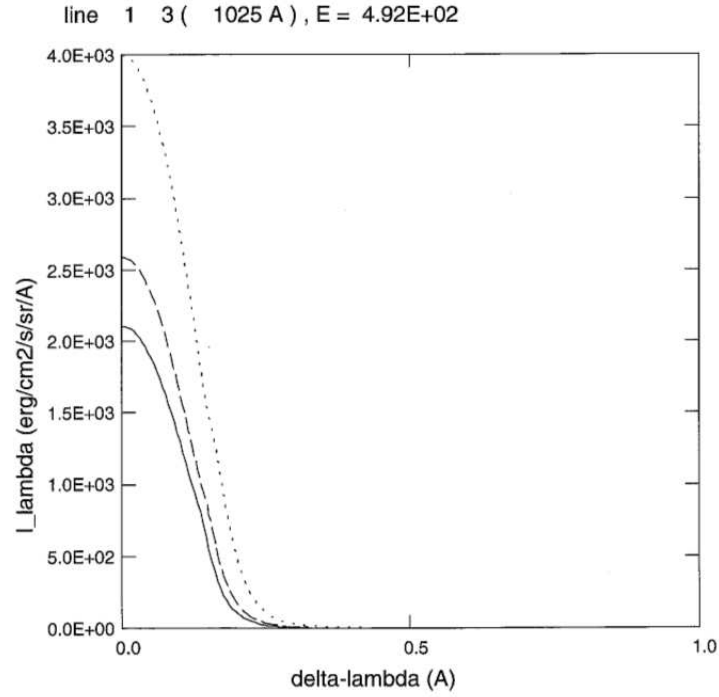


Figure 8: Half-profile of $L\beta$ line at 1025Å

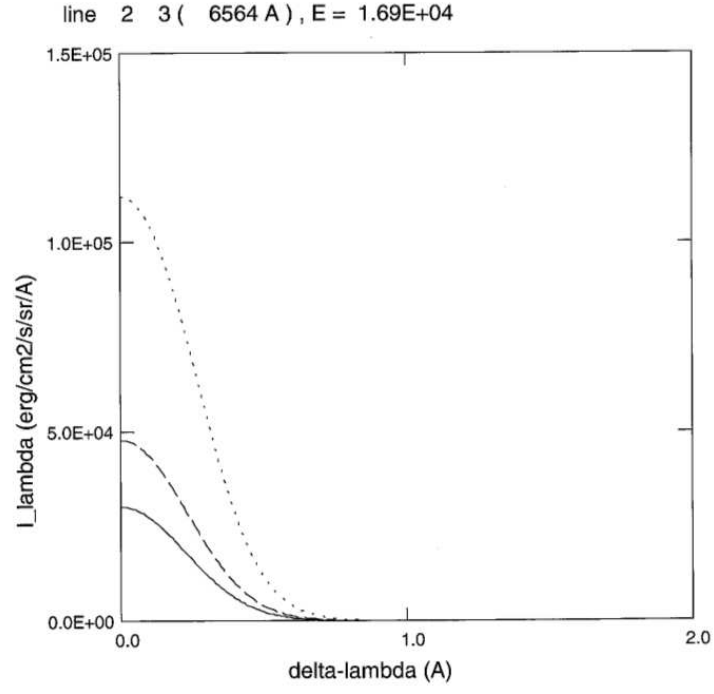


Figure 9: Half-profile of $H\alpha$ line at 6564Å

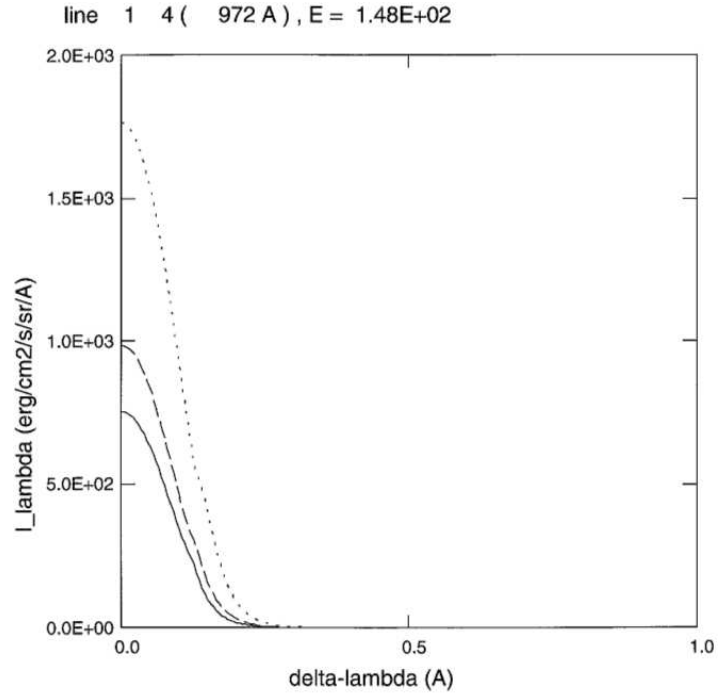


Figure 10: Half-profile of $L\gamma$ line at 972Å

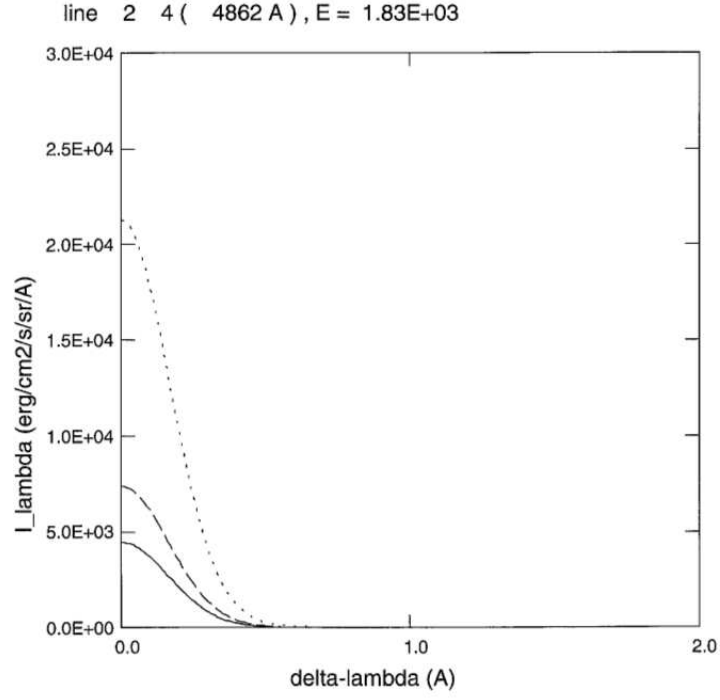


Figure 11: Half-profile of $H\beta$ line at 4862Å

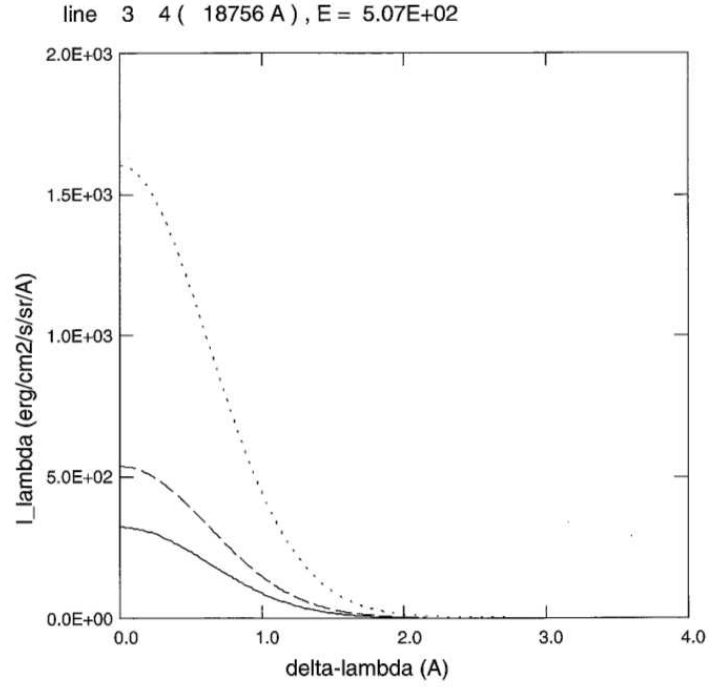


Figure 12: Half-profile of $Pa - \alpha$ line at 18756Å

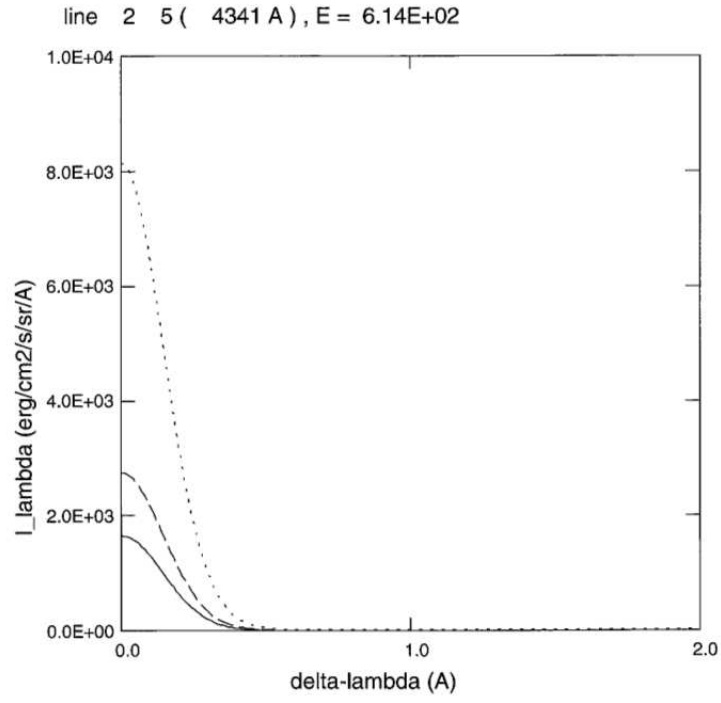


Figure 13: Half-profile of $H\gamma$ line at 4341Å

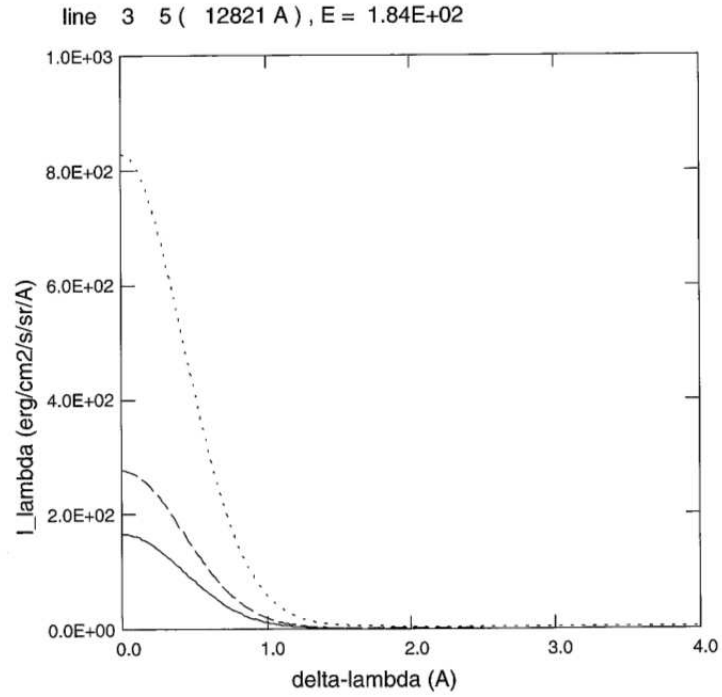


Figure 14: Half-profile of $Pa - \beta$ line at 12821Å

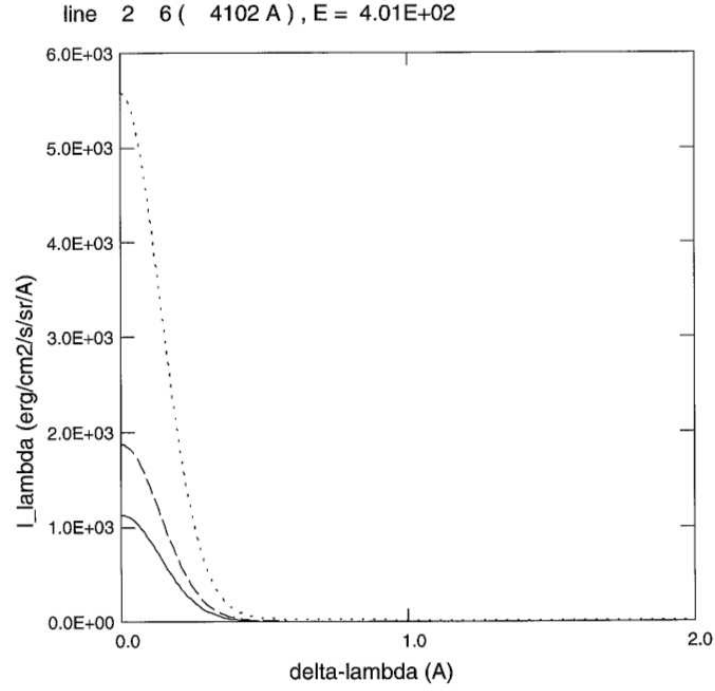


Figure 15: Half-profile of $H\delta$ line at 4102Å

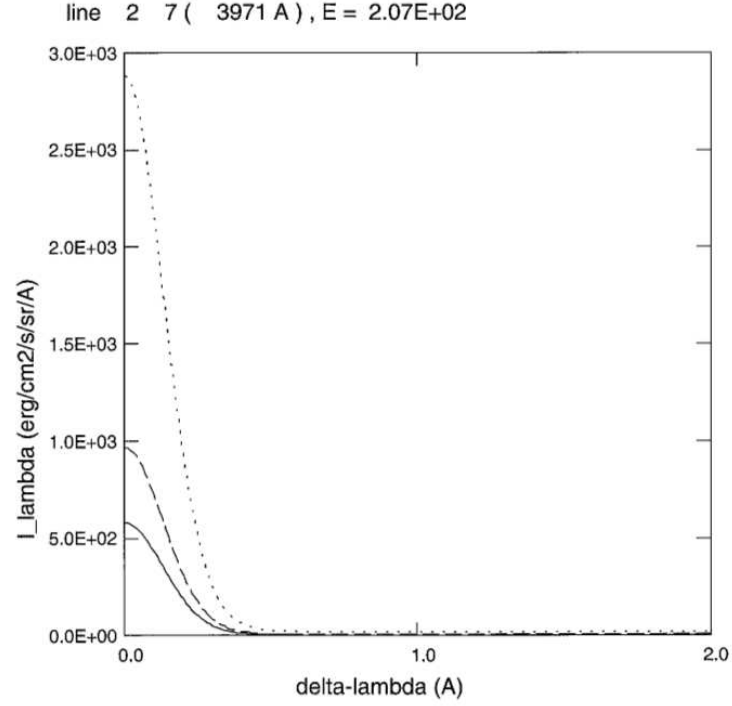


Figure 16: Half-profile of $H\epsilon$ line at 3971Å

11 Running PROM5

- Download the package PROM5-Licence-GPL.tgz from MEDOC website:
<https://idoc.osups.universite-paris-saclay.fr/medoc/tools/radiative-transfer-codes/>
- gfortran compiler is required.
- Unpack the package by typing the following linux command:
tar -xvzf PROM5.tgz
- Go to folder PROM5:
cd PROM5
- The folder contains the following files: intinc.dat, model5.dat, makefile, prom5.f90
- Run the code by typing:
make
./prom5
- The output files are: profil5.dat, profil5.ps
- The folder **results** contains the output files corresponding to a test case to be able to check if your results are good.

12 CPU time

On a PC with 4 Intel processors (2.67 *GHz*), the CPU time for PROM5 code with the prominence model defined in figures 5 and 6 (“model5.dat”) is: 31.698 *s* < 1 *min*.

13 Acknowledgements

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M. C-Y

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