

**Additional program to Coherent Structure Tracking
(CST) algorithm: computation of the divergence
from spherical velocities (V_φ , V_θ)**

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1 Introduction

Once the CST algorithm has computed the Solar surface velocities (e.g V_r , V_φ , V_θ on the (\vec{x}, \vec{y}) sky plan. θ and φ are the heliographic latitude and longitude) and their divergences in cartesian coordinates (see [the latest user guide from July, 2023](#)), an additional program, written in IDL, will calculate the divergence of the velocity field in spherical coordinates. This divergence, from V_φ and V_θ , are in 1/sec and are comparable to those obtained from helioseismology. The formulas implemented in this program are described in details in [section 3](#).

2 Running the additional program and test case

- Download the package source file `div_spher_CST.tgz` from [MEDOC](#) webpage
- SWIDL software is required. Type the following linux commands:
 - `tar -xvzf div_spher_CST.tgz`
 - `cd div_spher_CST`
- The directory contains input files (`latitude_HMI_586.fits`, `longitude_HMI_586.fits` and `vr_vtheta_vphi_all_2days_dec2022_deconv.sav`) corresponding to the following test case : HMI data (2 days) from December 28 and 29, 2022. These are output files from the CST algorithm (see figure 1 of [the latest user guide from July, 2023](#)). More precisely from step 1 for the computing of latitude and longitude, and from step 3 for the computing of velocities V_r , V_φ , V_θ in spherical coordinates on the sky plan or grid (\vec{x}, \vec{y}) .
- The directory also contains the IDL program (`calcul_div_spher_xyz_complet.pro`).
- With SWIDL, type the following command : `.r calcul_div_spher_xyz_complet.pro`
- `div_spheric_results.fits` contains the results (the divergence of the velocity field in spherical coordinates).
- Figure `IDL0.png` shows the spherical divergence at each time step.
- Figure `IDL1.png` shows the spherical divergence (averaged over 16 30-minute images, i.e. 8 hours) calculated from V_φ and V_θ given by CST algorithm (V_x, V_y, V_{Dop}).

3 Formulas implemented in the additional program

Here is a set of variables used in formulas. They are illustrated in [figure 1](#).

- $(\vec{x}, \vec{y}, \vec{z})$: Sun reference e.g the reference system in which we observe
- In our case, $V_r = 0$ because the flows are surface flows
- $M(x, y, z)$: a point on solar disk in Sun reference

- θ : heliographic latitude
- $\lambda = \frac{\pi}{2} - \theta$: heliographic colatitude
- φ : heliographic longitude
- V_λ, V_φ : velocity components at point M in Sun reference
- λ and φ depend on (x, y, z)
- R: Solar radius

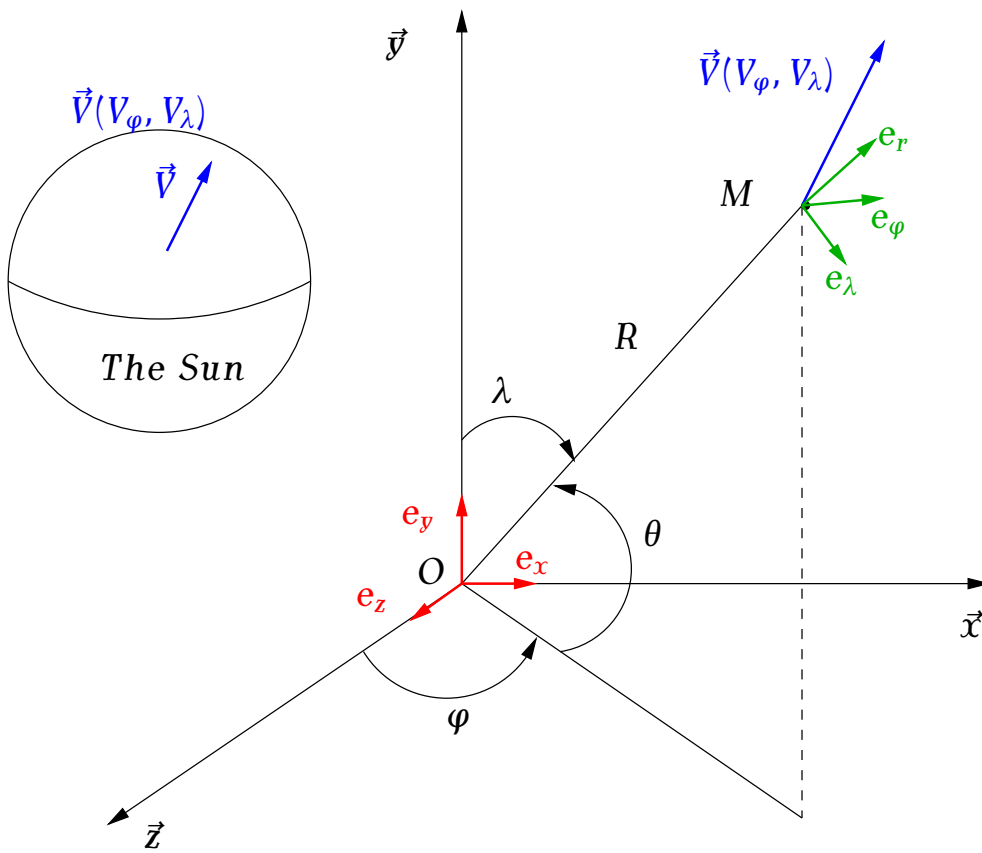


Figure 1: Coordinate systems

The divergence of the velocity field in spherical coordinates is written as:

$$\text{div}(\vec{V}) = \frac{1}{R \sin \lambda} \left[\sin \lambda \frac{\partial V_\lambda}{\partial \lambda} + \cos \lambda \cdot V_\lambda \right] + \frac{1}{R \sin \lambda} \frac{\partial V_\varphi}{\partial \varphi} \quad (3.1)$$

Let's write the relationship between dx , dy , $d\lambda$, $d\varphi$.

$$\text{As } \vec{OM} = R \cdot \vec{e}_r = R (\cos \lambda \cdot \vec{e}_y + \sin \lambda \cos \varphi \cdot \vec{e}_z + \sin \lambda \sin \varphi \cdot \vec{e}_x),$$

then

$$\begin{aligned} d\overrightarrow{OM} &= -R \sin \lambda \, d\lambda \cdot \vec{e}_y + R (\cos \lambda \cos \varphi \, d\lambda - \sin \lambda \sin \varphi \, d\varphi) \vec{e}_z \\ &+ R (\cos \lambda \sin \varphi \, d\lambda + \sin \lambda \cos \varphi \, d\varphi) \vec{e}_x \\ &= dx \cdot \vec{e}_x + dy \cdot \vec{e}_y + dz \cdot \vec{e}_z, \end{aligned}$$

thus

$$\begin{cases} dx = R (\cos \lambda \sin \varphi \, d\lambda + \sin \lambda \cos \varphi \, d\varphi) \\ dy = -R \sin \lambda \, d\lambda \\ dz = R (\cos \lambda \cos \varphi \, d\lambda - \sin \lambda \sin \varphi \, d\varphi). \end{cases} \quad (3.2)$$

We know that

$$\begin{cases} \frac{\partial V_\lambda}{\partial \lambda} = \frac{\partial V_\lambda}{\partial x} \frac{\partial x}{\partial \lambda} + \frac{\partial V_\lambda}{\partial y} \frac{\partial y}{\partial \lambda} + \frac{\partial V_\lambda}{\partial z} \frac{\partial z}{\partial \lambda} \\ \frac{\partial V_\varphi}{\partial \varphi} = \frac{\partial V_\varphi}{\partial x} \frac{\partial x}{\partial \varphi} + \frac{\partial V_\varphi}{\partial y} \frac{\partial y}{\partial \varphi} + \frac{\partial V_\varphi}{\partial z} \frac{\partial z}{\partial \varphi}. \end{cases} \quad (3.3)$$

According to (3.2), we have

$$\begin{cases} \frac{\partial x}{\partial \lambda} = R \cos \lambda \sin \varphi \\ \frac{\partial y}{\partial \lambda} = -R \sin \lambda \\ \frac{\partial z}{\partial \lambda} = R \cos \lambda \cos \varphi \\ \frac{\partial x}{\partial \varphi} = R \sin \lambda \cos \varphi \\ \frac{\partial y}{\partial \varphi} = 0 \\ \frac{\partial z}{\partial \varphi} = -R \sin \lambda \sin \varphi. \end{cases} \quad (3.4)$$

Then (3.3) can be written as

$$\begin{cases} \frac{\partial V_\lambda}{\partial \lambda} = R \cos \lambda \sin \varphi \frac{\partial V_\lambda}{\partial x} - R \sin \lambda \frac{\partial V_\lambda}{\partial y} + R \cos \lambda \cos \varphi \frac{\partial V_\lambda}{\partial z} \\ \frac{\partial V_\varphi}{\partial \varphi} = R \sin \lambda \cos \varphi \frac{\partial V_\varphi}{\partial x} - R \sin \lambda \sin \varphi \frac{\partial V_\varphi}{\partial z}. \end{cases} \quad (3.5)$$

Thanks to (3.5), the divergence of the velocity field (3.1) can then be written as

$$\begin{aligned} \operatorname{div}(\vec{V}) &= \cos \lambda \sin \varphi \frac{\partial V_\lambda}{\partial x} - \sin \lambda \frac{\partial V_\lambda}{\partial y} + \cos \lambda \cos \varphi \frac{\partial V_\lambda}{\partial z} \\ &+ \frac{V_\lambda}{R \tan \lambda} + \cos \varphi \frac{\partial V_\varphi}{\partial x} - \sin \varphi \frac{\partial V_\varphi}{\partial z} \end{aligned} \quad (3.6)$$

Due to the orientation of angles θ and λ , we have $V_\lambda = -V_\theta$ and $d\theta = -d\lambda$.

$$\text{Then } \frac{\partial V_\lambda}{\partial x} = -\frac{\partial V_\theta}{\partial x}, \quad \frac{\partial V_\lambda}{\partial y} = -\frac{\partial V_\theta}{\partial y}, \quad \frac{\partial V_\lambda}{\partial z} = -\frac{\partial V_\theta}{\partial z}.$$

The divergence of the velocity field in spherical coordinates can be written as follows

$$\begin{aligned} \operatorname{div}(\vec{V}) &= -\sin \theta \sin \varphi \frac{\partial V_\theta}{\partial x} + \cos \theta \frac{\partial V_\theta}{\partial y} - \sin \theta \cos \varphi \frac{\partial V_\theta}{\partial z} \\ &+ \cos \varphi \frac{\partial V_\varphi}{\partial x} - \sin \varphi \frac{\partial V_\varphi}{\partial z} + \frac{\tan \theta}{R} V_\lambda \end{aligned} \quad (3.7)$$

The term $\frac{\tan \theta}{R} V_\lambda$ is negligible because R is large.

Note that the set of equations (3.2) can be written as

$$\begin{cases} \partial x = -R \sin \theta \sin \varphi d\theta + R \cos \theta \cos \varphi d\varphi \\ \partial y = R \cos \theta d\theta \\ \partial z = -R (\sin \theta \cos \varphi d\theta + \cos \theta \sin \varphi d\varphi) \end{cases} \quad (3.8)$$

which are implemented in the IDL program "calcul_div_spher_xyz_complet.pro".