Measure of Solar surface velocities from granule tracking: Coherent Structure Tracking (CST) algorithm.

ANNEX VERSION 1.2

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1 Introduction

Coherent **S**tructure **T**racking (**CST**) algorithm (Roudier et al. (2012); Švanda et al. (2013); Roudier et al. (2013); Rincon et al. (2017); Roudier et al. (2018)) is a set of codes written in IDL and Fortran 90, computing the horizontal velocity field on the Sun surface by using solar granules as tracers. CST codes take as input HMI/SDO intensity images (hmi.Ic_45s), with a time step of 45s. Each record includes a list of keywords and one image of the Sun (2D, 4096x4096) in FITS format. We use also the HMI/SDO Dopplergrams (hmi.V_45s), to derive spherical velocities on the Sun surface. The Vx and Vy (in km/s) are computed at a cadence of 30 min with a spatial window of 7 pixels, equivalent to 3.5 arcsec, around 2.5 Megameters (Mm) (Rieutord et al., 2001). Original HMI/SDO data have the North at the bottom and East on the right of the image; the images are rotated in order to have the North on top and East on the left. To read and treat HMI/SDO data, we use SSWIDL and ifort compiler. The Fortran part is parallelized with OpenMP.

Annex version 1.2 is spatial and temporal U_x and U_y (velocity) resolution improvements and is available from the MEDOC. That CST application uses as input HMI/SDO intensity images (hmi.Ic_45s), with a time step of 45s (file size of each data is 4096x4096 pixels) which are deconvolved by an IDL code (output intensity file size is 8192x8192 pixels) before running the "deconvolved" Fortran part. Velocity (U_x and U_y) on all the Sun for a temporal window of 30 min are obtained. This annex version 1.2 is described in details in section 10.

From section 2 to section 9, we explain in details original CST codes (**Version 1.1 of CST codes** is available from the MEDOC website) : downloading of HMI intensity and Doppler data from JSOC (HMI 45s-cadence data series are not available from MEDOC); data reduction; algorithm; etc. It is necessary to understand how the original version of CST codes (version 1.1) works before using annex version 1.2. Furthermore, formulas implemented in original CST codes (Versions 1.0 and 1.1) are described in details in section 7.

The formulae for solar derotation applied to Doppler and intensity data are described in section 7.1.4.

In this updated documentation, we describe how to use CST codes on deconvoluted data (see sections 5 and 6).

2 Description of CST codes

Nature of the physical problem: Measure of Solar surface velocities

Method of solution: Granule tracking, Daubechies wavelets

Other relevant information: IDL step 3 is reduced to a single file instead of 3 files in the previous version of CST codes (V1.0)

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Program available from: https://idoc.osups.universite-paris-saclay.fr/medoc/tools/cst-codes/

Version: **1.1**

Computer(s) on which program has been tested: IAS server

Operating System(s) for which version of program has been tested: Linux (Debian 10)

Programming language used: IDL (with **SSWIDL** software) and Fortran 90 (with **ifort** compiler)

Status: Stable

Accessibility: open (MEDOC)

Nb. of code lines in combined program and test deck: 3111 lines for IDL files and 4018 lines for Fortran files (librairies not included)

Typical running time: for 30 min HMI observations, IDL parts take 15 min and Fortran part takes 1h16 on IAS server with the following characteristics (CPU: 2 x Intel(R) Xeon(R) CPU E5-2650 v4 @ 2.20GHz 24 cores, Memory: 256Go)

3 CST algorithm

Figure 1 describes the scheme of CST codes.

They take as input HMI/SDO instensity images and Dopplergrams, with a time step of 45s (4096 x 4096 pixels). The procedure to take HMI intensity and Doppler data from JSOC is described in sections 4 and 5. CST codes can be run up to 6 HMI observation days. The reference day is the 4th observation day (for more details, see remark 3.2).

Remark 3.1 Definition of derotation
We often use the term "derotation" : this term indicates that we have applied an algorithm that corrects natural differential rotation of the Sun and brings back longitudes related to solar surface at the same locations for each time deviation from the origin of the first HMI image. In other words, points of the longitude 0 degrees at t=0 which are at different longitudes are brought back to t= 10 min for example at longitude 0 degrees for this time. Thus we can follow the evolution in time of the structures at their original longitude.

CST algorithm is divided into 3 steps :

- step 1 : CST IDL part

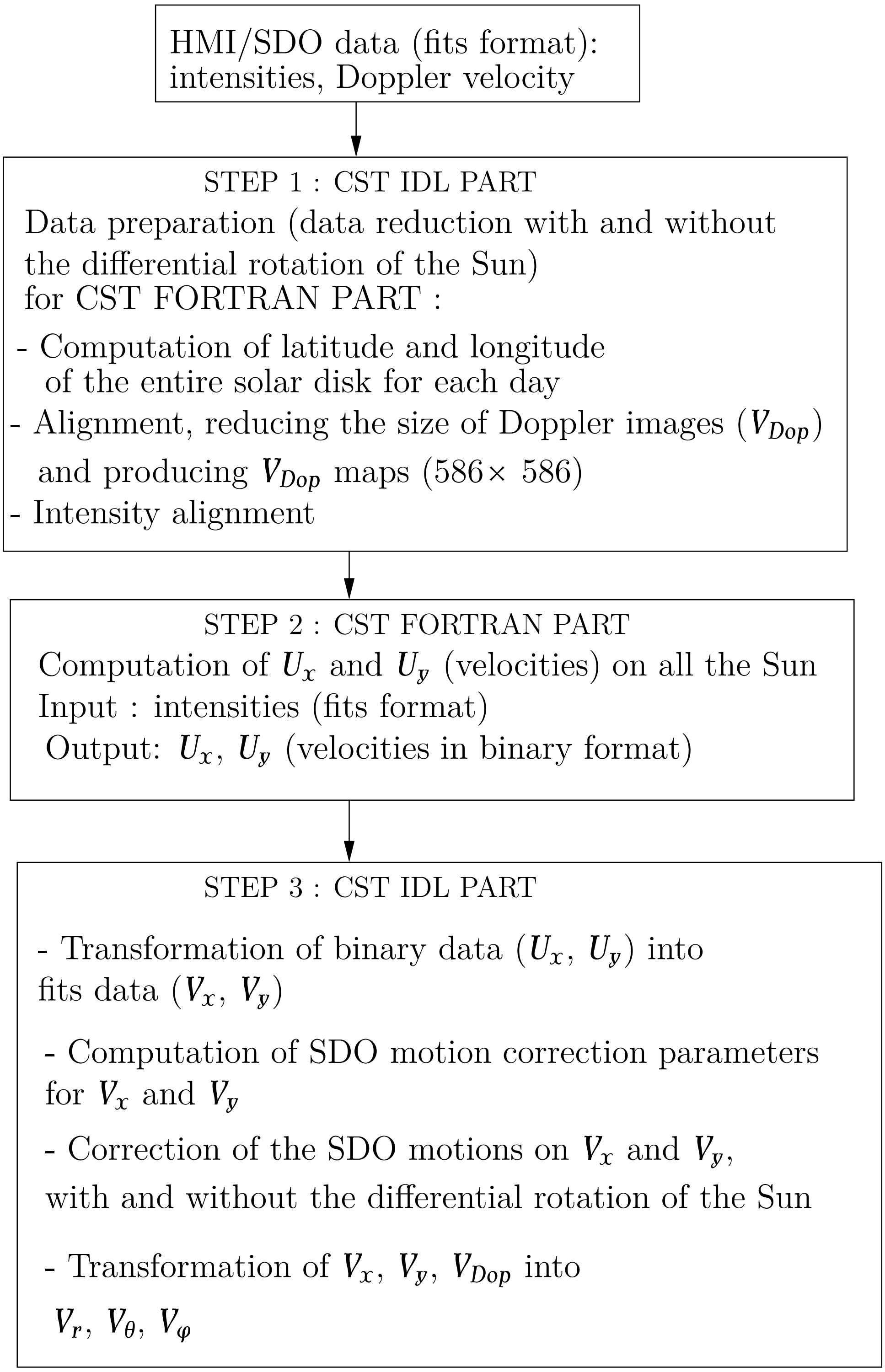

Figure 1: Scheme of CST codes

- step 2 : CST Fortran part
- step 3 : CST IDL part

We describe below each step of CST codes.

3.1 Step 1 : CST IDL part

The first step concerns data preparation under IDL using SSWIDL as input for CST Fortran part (step 2). More precisely, it is data reduction with recentring and (with or not) derotating of the differential rotation of the Sun. It summarized as follow :

- computation of latitude and longitude of the entire solar disk for each observation day
- alignment and resizement of the Doppler data. In output files, North is at the bottom and file size is 4096 x 4096 pixels
- SDO motion correction : we correct satellite motion on Doppler data. In output files, North is at the top and the file size is 4096 x 4096 pixels
- limbshift correction : we measure the limbshift; we calculate circular average and we correct the Doppler data for each pixel of the Sun. In output files, North is at the top and file sizes are 586 x 586 pixels and 4096 x 4096 pixels
- average over 30 min of Doppler data: available file V_{Dop} with solar rotation. In output files, North is at the top and file size is 586 x 586 pixels
- measure of the solar rotation from Doppler data (or use the standard solar rotation)
- derotation of Doppler data (see figure 2). In output files, North is at the top and file size is 586 x 586 pixels
- average over 30 min of Doppler data: we produce V_{Dop} maps without solar rotation. In output files, North is at the top and file size is 586 x 586 pixels
- intensity alignment : recentering and derotating in the same way as before for Doppler data,
  - ⋆ with rotation (aligned and resized, North top)
  - ⋆ derotation of the intensity (North top)

  The output file size is 4096 x 4096 pixels.

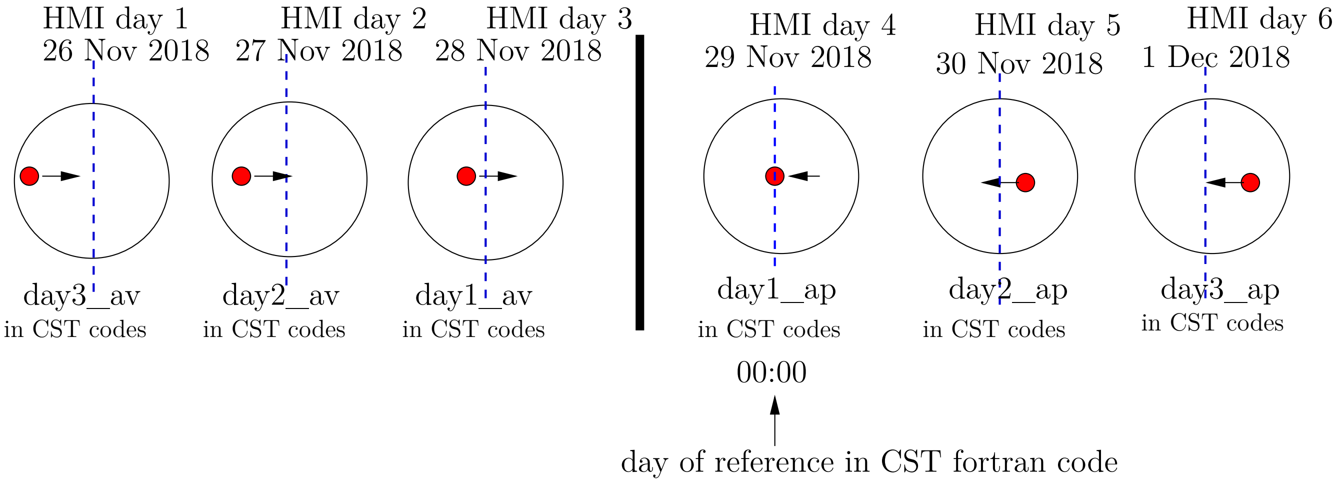

Figure 2: Derotation of Doppler data.

Remark 3.2 *CST codes can be run up to 6 (HMI observation) days : the reference day for CST Fortran code is the 4th observation day (the reference of the center of the Sun and the solar radius are taken at 00:00 of the 4th observation day). This remark has a sense when running CST codes (after step 1, data reduction). For more details, see section 5. Figure 8 illustrates, as example, 6 days from November 26 to December 1, 2018.*
To run CST codes for 4 observation days, the 2nd observation day is the reference day. Figure 6 illustrates, as example, 4 days from November 28 to December 1, 2018.
Figures 7, 5, 4 and 3 illustrate respectively, as example, 5 days, 3 days, 2 days and 1 day.

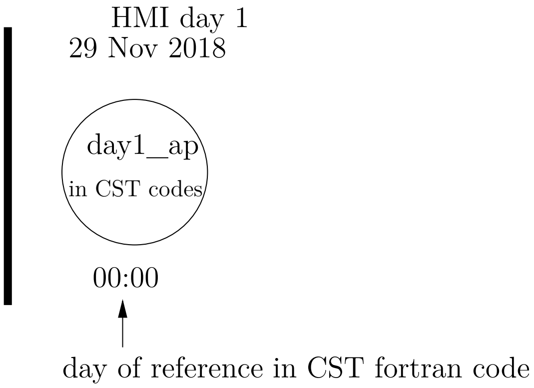

Figure 3: CST run for 1 observation day. The observation day (November 29, 2018) is the reference day.

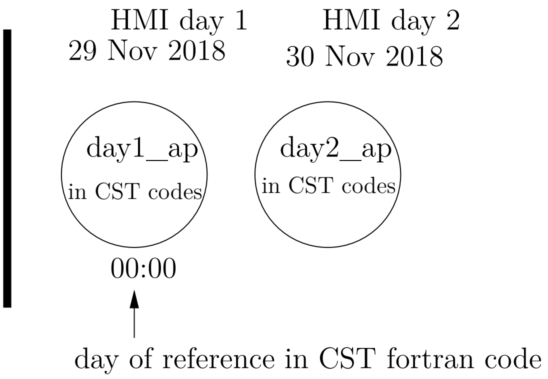

Figure 4: CST run for 2 observation days. The 1st day (November 29, 2018) is the reference day.

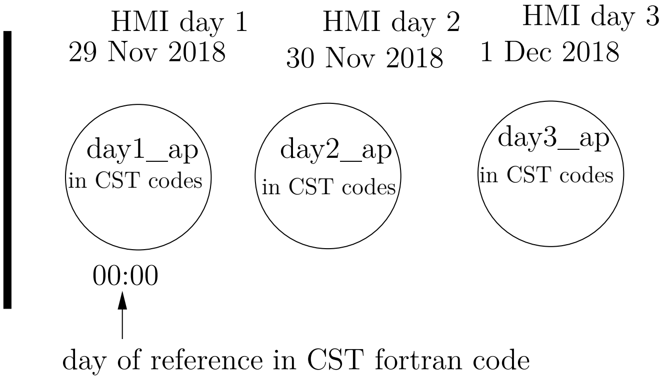

Figure 5: CST run for 3 observation days. The 1st day (November 29, 2018) is the reference day.

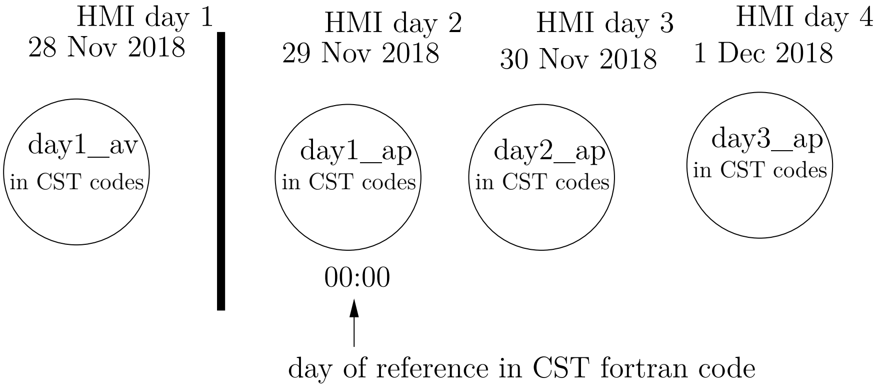

Figure 6: CST run for 4 observation days. The 2nd day (November 29, 2018) is the reference day.

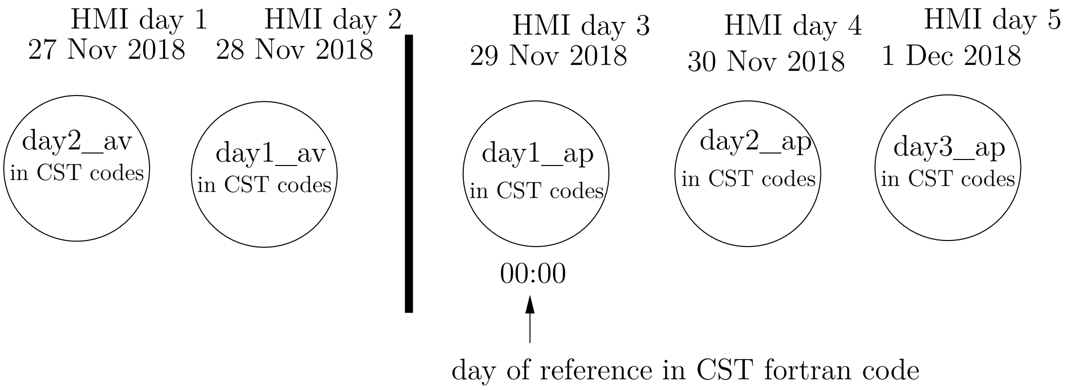

Figure 7: CST run for 5 observation days. The 3rd day (November 29, 2018) is the reference day.

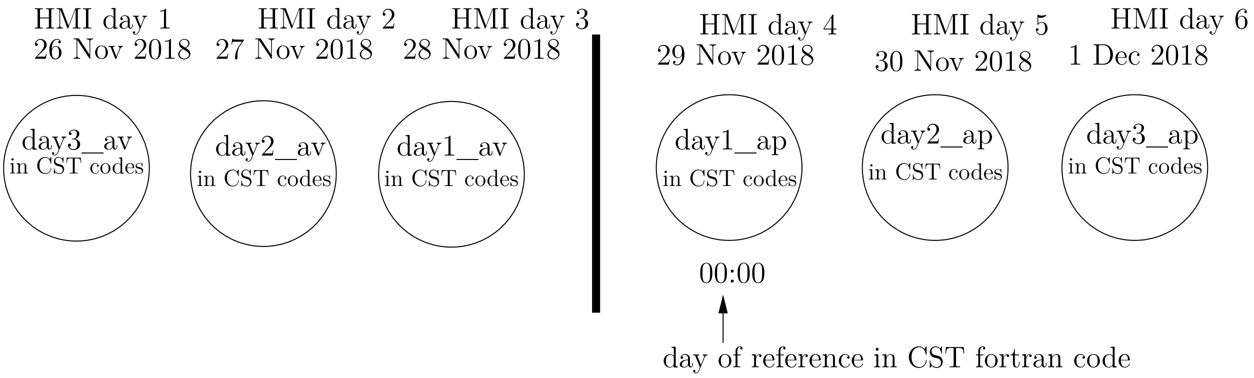

Figure 8: CST run for 6 observation days. The 4th day (November 29, 2018) is the reference day.

3.2 Step 2 : CST Fortran part

The second step computes the velocities (U_x and U_y) on all the Sun for a temporal window of 30 min from intensity images. It uses CST Fortran code. Two possible choices : data with solar rotation or data without solar rotation.

The principle of this step (of the CST) is to follow granules in order to define their lifetime and coherence in time. More specifically :

- assume that granules are objects advected by an underlying flow which we wish to measure
- the lifetime of a coherent object (granule) is defined between its appearance and disappearance
- if granule splits : the life is stopped and children = new granules (see t_3 and t_4 on figure 9)

- if granules merge : the lives of granules that merge are stopped and the new granule issued from merging = new granule (see t_1 and t_2 on figure 9).

We can follow coherent structures between their birth and death (see figure 9).

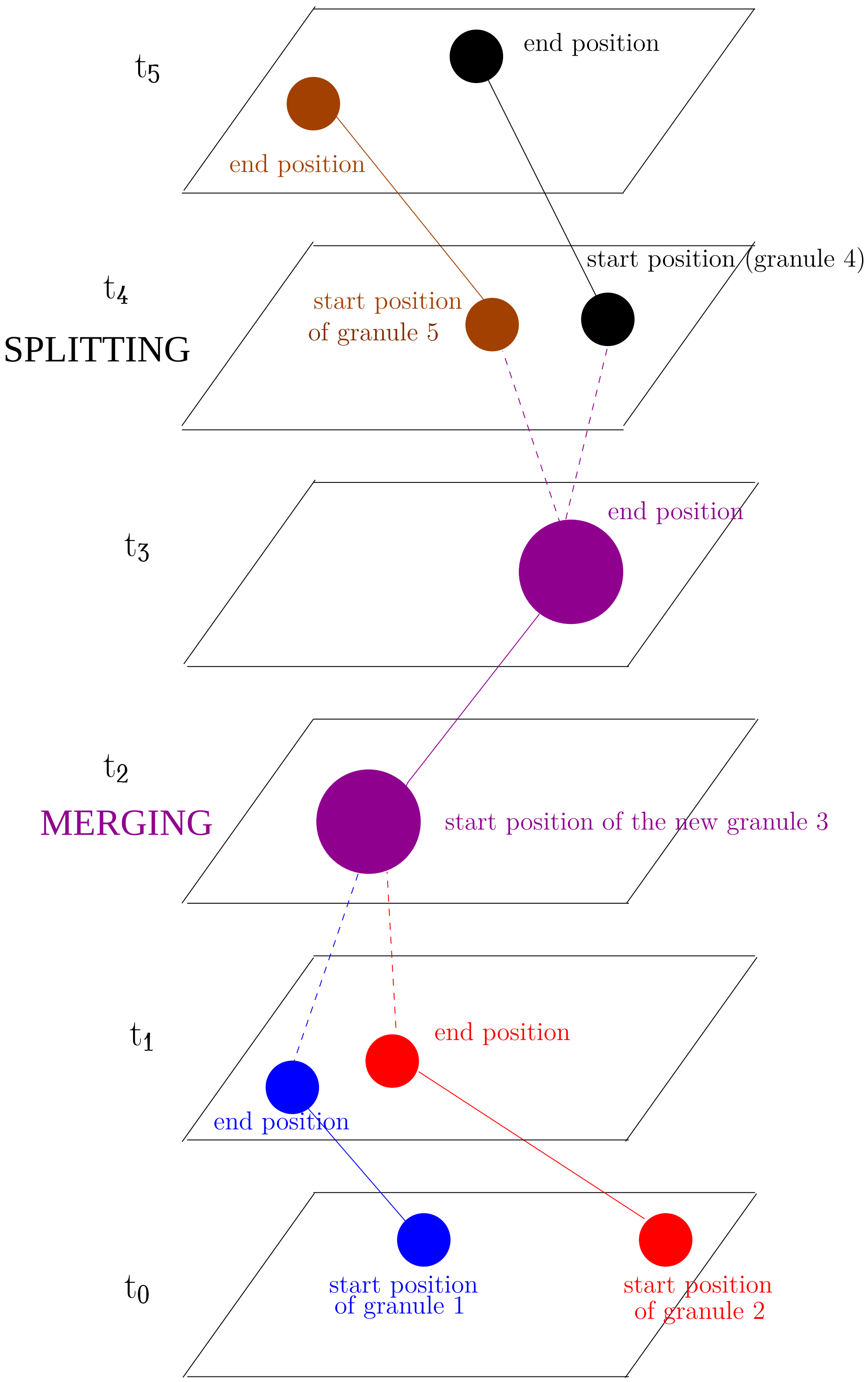

Figure 9: Step 2 (CST) principle.

During their lifetime, granules can split or merge into multiple objects. Disappearence or appearance between two successive frames must be taken into account. Then, the life of coherent objects (i.e granules) is defined between its appearance and disappearance if the granule does not split or merge. When the granule splits, the life of granule is stopped and its children are considered as new granules. In the same way, when granules merge, the lives of the granules that merge are stopped and the new granule issued from the merging is considered as a new granule. Thus, we can follow a coherent structure between their birth and death.

For each detected granule, velocity between its start position and end position is measured. Velocities which are in spatial windows 2,5 Mm× 2,5 Mm and over 30 minutes (temporal windows) are averaged (Roudier et al. (1999), Roudier et al. (2012)).

3.3 Step 3 : CST IDL part

The third step uses IDL codes to transform velocity binary data (U_x and U_y) into fits data (V_x and V_y); to compute SDO motion correction parameters on V_x and V_y; to correct SDO motions on V_x and V_y with and without the differential rotation of Sun; to transform V_x, V_y, V_{Dop} into V_r, V_φ, V_θ (with and without rotation).
Figure 10 is an example of results provided by step 3 of CST codes. It represents the module of (V_φ,V_θ) obtained for three hours of HMI data, where supergranulation is visible.

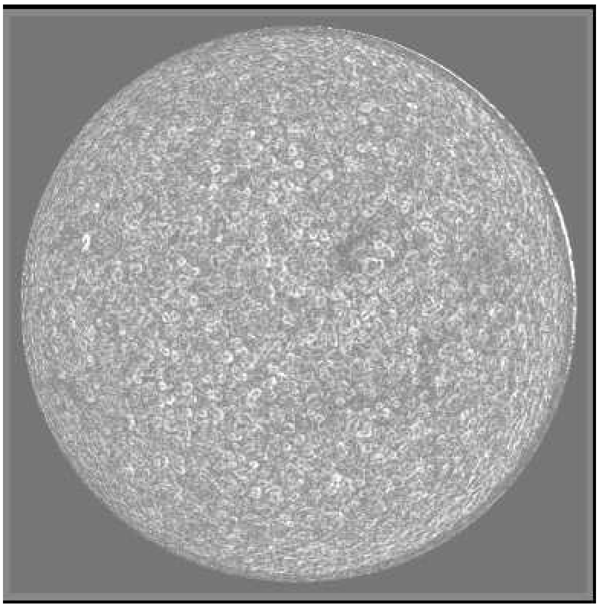

Figure 10: Module of (V_φ,V_θ) for three hours of HMI data (step 3).

Formulas used in IDL and Fortran codes can be found in Rincon et al. (2017).

In section 4, we explain how to take HMI/SDO data from JSOC. In section 5, we explain how to run CST software. In section 6, we describe in details input file to modify. In section 8, we give an example of CST software run applied to 30 min HMI observations.

4 Procedure to take SDO/HMI data from JSOC

45s-cadence HMI files are not available at MEDOC.

An automatic procedure to take SDO/HMI data from JSOC is proposed by T. Corbard (OCA). “codes_data_JSOC” directory contains 3 codes (exportfile.csh, url_escape.pl, gethmi_cst.pro) to run in order to download automatically SDO/HMI data (29 November, 2018, 8:00 to 8:30, for example).

In order to get the HMI data in the directory /data/cluster/workshop_CST2020/day1_apres/, type the following commands in the directory which contains the three codes (exportfile.csh, url_escape.pl, gethmi_cst.pro):

1. `chmod +777 exportfile.csh`
2. Then with IDL :

   ```
   > .r gethmi_cst.pro
   > gethmi_cst,’your_registered_JSOC_email_adress’,date_s=[11,29,2018,8,0],date_e=[11,29,2018,8,30],out_dir=’/data/cluster/workshop_CST2020/day1_apres/’,/exec
   ```

For the brave ones, here is the manual procedure to take HMI data directely from JSOC website. But you can see that automatic procedure introduced above is certainly the fastest one.

1. On the JSOC webpage, click on data access and then in look data (top of the page)
2. click on Fetch seriesname list
3. select hmi.Ic - continuum intensities with a cadence of 45s
4. write for example 30 min on November, 29, 2018:

   hmi.Ic_45s[2018.11.29_08:00_TAI-2018.11.29_08:30_TAI]

   or for one day: hmi.Ic_45s [2018.11.29/1d].

   See examples in the web page for different requests
5. click on GetRecordCount (this gives you the number of files requested)
6. click on Export Data (top of the page)

7. click on Export (on the right)
8. click on method and get url-tar
9. fill the notify with your email address. An email will be sent to you to confirm your registration
10. fill Requestor with your name
11. click on Check Params for Export
12. when green, click on Submit Export Request
13. an email will be sent to you with the link for downloading the data in tar format: JSOC_20191104_695.tar. Click on tar file link to download on your PC or type the linux command to get results from a server, for example:

    ```
    wget http://jsoc.stanford.edu/SUM85/D1231699647/S00000/JSOC_20191104_695.tar
    ```

    Type the following linux command:

    ```
    tar -xvf JSOC_20191104_695.tar
    ```

    You get 41 files from hmi.Ic_45s.20181129_080000_TAI.2.continuum.fits
    up to hmi.Ic_45s.20181129_083000_TAI.2.continuum.fits.

    For 1 (HMI observation) day, we get 1920 files (one file each 45 seconds)
14. repeat operation at point 4 to download Doppler hmi.V_45s - Dopplergrams with a cadence of 45s.

Data size for 30 min HMI observations (intensity and Doppler) is 2.7 G.
Data size for 1 day HMI observations (intensity and Doppler) is 63 G.

Moreover, deconvoluted data can be used due to higher quality. Indeed, with these deconvoluted data, we can detect more granules, giving us a better measure of velocities at the sun's surface. To get these deconvoluted files in the JSOC data base, choose:

hmi.Ic_45s_dcon[***] ; intensity
hmi.V_45s_dcon[***] ; Doppler
hmi.M_45s_dcon[***] ; Magnetic field.

It is necessary to modify these two parameters in the "param_seq_29nov2018_EOS_30mn" file (see section 6) such as:
ngmax0=960000 : maximum number of granules per image
ngmax = 29000000 : maximum number of granules tracked during the 30-minute sequence

5 Running CST

- Download the package source file CST_V1.1.tgz from MEDOC webpage
- ifort compiler and SSWIDL software are required. Type the following linux commands:
- tar -xvzf CST_V1.1.tgz
- cd CST_V1.1
- The directory contains 1 directory (codes_data_JSOC) for taking automatically HMI data from JSOC and 2 directories corresponding to 2 different cases: a test case (29 Nov 2018 HMI observations of 30 min) to be able to check if your results are good (“CST_TEST_30min” directory), source codes to treat from 1 to 6 observation days (“CST_1_TO_6_DAYS” directory, adapted for the day 29 Nov 2018 HMI observations).
- Each of these directories (corresponding to the 2 different cases above) contains the following files:

  1. Fortran files (“codes_CST_Fortran” directory):
     - cst_labv7_FS_2017.f90 (main program)
     - Makefile.common (used to compile librairies that are in the Lib directory)
     - deriw2d.f90
     - detect.f90
     - div_curl.f90
     - mres2d.f90
     - opt_scale.f90
     - the_name.f90 (contains thename module)
     - segment.f90 (contains segmentation module)
     - cg.f90
     - interpol2d.f90
     - interpol2dth.f90
     - interpol2d_routine.f90 (contains interpolationroutine module)
     - interpol2d_routine_short.f90
     - centre_gravite.f90
     - input.f90 (contains input subroutine)
     - sub_coef_r.f90
     - fftw3.inc, form_num.inc, Makefile.inc
     - “lib” directory contains librairies needed for compilation
     - Makefile (file to modify): variables CFLAGS and CFLAGS_LIB are to be adapted according to your server (-I/usr/include/x86_64-linux-gnu/c++/8)
     - script_29nov2018.sh (file to modify): according to your server, you need to add or not the following commands (see section 6 for more details) :
     “source /opt/intel/compilers_and_libraries_2018.3.222/...” : path to Fortran ifort compiler
     “ulimit -s unlimited”
     - param_seq_29nov2018_EOS_30mn (file to modify) :

the following parameters are to be adapted according to the HMI/SDO data: prefix, input_file, arcsec, pixel (see section 6 for more details)

2. IDL files (codes_CST_idl directory):
   - reduction_doppler_intensity_all_days_apres.pro
   - reduction_doppler_intensity_all_days_avant.pro
   - step3_CST_IDL_apres.pro
   - step3_CST_IDL_avant.pro
   - image_cont.pro : to read the binary file "image_cont" (see STEP 2)

- take HMI intensity and Doppler data from JSOC (45s-cadence HMI files are not available at MEDOC). For more details, see section 4.
- CST codes can be run up to 6 days (cf "CST_1_TO_6_DAYS" directory): the reference day is the 4th day (the reference of the center of the Sun and the Solar radius are taken at 00:00 of the 4th day). More precisely, for 6 days, transfer the intensity and Doppler data in each directory corresponding to the day of the sequence:

  - "day3_avant" directory: 1st day data
  - "day2_avant" directory: 2nd day data
  - "day1_avant" directory: 3rd day data
  - "day1_apres" directory: 4th day data (reference day)
  - "day2_apres" directory: 5th day data
  - "day3_apres" directory: 6th day data

  First, you must create these directories and second, put data inside.

  Note : To run CST codes for 5, 4, 3, 2 and 1 days, see remark 3.2

- For the test case of 30 min HMI observations, you must create "day1_apres" directory and put the HMI data (intensity and Doppler) inside.
- Create the result directorys "treated_day1_ap" for 1 observation day, "treated_day1_ap" and "treated_day2_ap" for 2 observation days, ...
- Adapt the following parameters in reduction IDL files (file name starting with "reduction"):
  path, path_out, nombre_j, rota (=0 for standard rotation (quiet Sun) or =1 for rotation measured directly on the data), path for tmp (temporary directory) and check that tmp directory has free space in memory
- **STEP 1: CST IDL PART**

  To prepare input data for CST Fortran program, type the following commands with SSWIDL for the test case (30 min HMI observations), in "codes_CST_IDL" directory:

1. .r reduction_doppler_intensity_all_days_apres.pro
2. .r reduction_doppler_intensity_all_days_avant.pro (depending of the treated case)

For 1 to 6 days HMI, type the following commands with SSWIDL:

1. .r reduction_doppler_intensity_all_days_apres.pro
2. reduction_doppler_intensity_all_days_apres.pro,nombre_j,rota
3. .r reduction_doppler_intensity_all_days_avant.pro (depending of the treated case)
4. reduction_doppler_intensity_all_days_avant.pro,nombre_j,rota (depending of the treated case)

The order in which the IDL codes are run is important: the reference day is chosen in the first code "reduction_doppler_intensity_all_days_apres.pro".

For example, for 1 observation day, the output files are (in FITS format) in "treated_day1_ap directory":

- co_latitude_HMI_4096.fits
- co_latitude_HMI_586.fits
- day1_apres_int_derot_000.fits, ..., day1_apres_int_derot_1909.fits (inputs for CST Fortran code)
- Doppler_derot_30mn.dat
- Doppler_derot_raw_0001.fits, ..., Doppler_derot_raw_1909.fits
- Doppler_derot_smooth_0001.fits, ..., Doppler_derot_smooth_1910.fits
- Doppler_limb4096_0001.fits, ..., Doppler_limb4096_1910.fits
- Doppler_raw_0001.fits, ..., Doppler_raw_1910.fits
- Doppler_smooth_0001.fits, ..., Doppler_smooth_1910.fits
- Doppler_with_rotation.dat
- latitude_HMI_4096.fits
- latitude_HMI_586.fits
- longitude_HMI_4096.fits
- longitude_HMI_586.fits
- SDO_Dop_cormvt_0001.fits, ..., SDO_Dop_cormvt_1910.fits
- SDO_seq_doppler_0001.fits, ..., SDO_seq_doppler_1910.fits
- SDO_seq_int_avec_rot_0001.fits, ..., SDO_seq_int_avec_rot_1909.fits

If there are not 1920 files (corresponding to 1 day), it is necessary to complete from the last (copy the last file). So we have a measure of the last half hour with a lower amplitude.

For 30 min HMI observations, we have:
- ...
- day1_apres_int_derot_000.fits, ..., day1_apres_int_derot_41.fits

- ...

For 6 days HMI observations, we have:
- ...
- day1_apres_int_derot_000.fits, ..., day1_apres_int_derot_1909.fits
- day2_apres_int_derot_000.fits, ..., day2_apres_int_derot_1909.fits
- day3_apres_int_derot_000.fits, ..., day3_apres_int_derot_1909.fits
- day1_avant_int_derot_000.fits, ..., day1_avant_int_derot_1909.fits
- day2_avant_int_derot_000.fits, ..., day2_avant_int_derot_1909.fits
- day3_avant_int_derot_000.fits, ..., day3_avant_int_derot_1909.fits
- ...

If there are not 1920 files for each day, it is necessary to complete from the last (copy the last file).

Warning: Data reduction uses the same "temporary" directory (/tmp) when reading SDO data. So you should not run the IDL code for 2 data sets at the same time, otherwise the SDO decompression files (in /tmp directory) will be destroyed by the 2nd IDL run.

- **STEP 2: CST FORTRAN PART**

Choose 18 cores for an optimal run. To compile and execute Fortran files, here are the commands for SLURM, in "codes_CST_Fortran" directory:

```
sbatch script_20nov2018.sh
```

The output files are for each 30 mn (in "JOB_XXXX/results" directory) :

- output.log: to check if CST code runs well
- image_cont: last segmented image of the Sun in binary format (visualized by an IDL program "image_cont.pro"(with SSWIDL : .r image_cont.pro))
- param_seq_ddmmyyyy_EOS_30mn: parameters used
- traject_11_0000: trajectories of all selected granules. he second number of the first line is the total number of treated granules. Column 1 is the granule number, column 2 is x_cent (gravity center), column 3 is y_cent, column 4 is the number of the image where the granule is born, column 5 is the number of the image where the granule dies, column 6 is the lifetime of the granule in second, column 7 is the velocity ux (in km/s), column 8 is the velocity uy (in km/s)
- nb_gran_0000: pixel size chosen for the spatial window, 1 arcsec in km, pixel size in arcsec, treatment threshold of CST code
- ux_b_0000, ux_h_0000, ux_l_0000, ux_m_0000, ux_k_0000
- uy_b_0000, uy_h_0000, uy_l_0000, uy_m_0000, uy_k_0000

- div_b_0000, div_h_0000, div_l_0000, div_m_0000, div_k_0000
- rot_b_0000, rot_h_0000, rot_l_0000, rot_m_0000, rot_k_0000
- err_b_0000, err_h_0000, err_l_0000, err_m_0000, err_k_0000
- ux,uy,div,rot, sampled on a regular grid , traj contains the trajectories of each granule.

ux_b_0000 file is for the first 30mn. The last 0 refers to time in "time window unit" (and following ux_b_0001 for the following 30min etc. The last 1 refers to time in "time window unit").

- raw: b (brut) raw ux and uy from CST (586 x 586 pixels)
- high: h: first high resolution wavelet filter (Daubechies)
- middle: m second wavelet filtering (Daubechies) half resolution of "h"
- large: l third filtering by wavelet (Daubechies) resolution half of "m" p
- extra-large: k fourth filtering by wavelet (Daubechies) resolution half "l"

For example the most common with the SDO data the resolution will be (if we take bin_sp = 7 (pixels) h = 2.5 Megameters (Mm), m = 5.0 Mm, l = 10Mm, k = 20 Mm output file speeds (ux, uy) on a size of 586x586 pixels, latitude, longitude and Doppler files are also set to this size and directly superimposable.

In the Fortran main program (cst_labv7_FS_2017.f90) which calculates horizontal speeds, a circular mask is used to properly zero the data outside the Sun. This mask is fixed once and for all in the program with the following parameters in pixels:
R$_{sol}$=1873 x$_{cent}$=2054.47 y$_{cent}$=2048.21.
As the diameter of the Sun changes with time, it is possible that in some times the mask is a little bit smaller than the solar disk. We can so modify R$_{sol}$, x$_{cent}$, y$_{cent}$ if this needed. Anyway even with a mask smaller, the program output V$_x$ and V$_y$ speeds are surimposable to Doppler and latitude and longitude grids in 586x586 pixels.

- **STEP 3: CST IDL PART**

1. Copy all ux and uy files from the "JOB_XXXX/results" directory to the "treated_day1_ap" directory. These files are needed as input data for IDL codes above

2. Adapt the following parameters in IDL files (in "codes_CST_IDL" directory):

   - step3_CST_IDL_apres.pro : path (path to original fits HMI files), path_out (path to result directory : treated_dayX_ap), path_vit (directory where are velocities ux, uy with and without rotation. Generally, it is the same path as path_out), derot (=1 : velocities without solar rotation; = 0 : velocities with solar rotation), nombre_j (number of days)
   - step3_CST_IDL_avant.pro : path (path to original fits HMI files), path_out

(path to result directory : treated_dayX_ap), path_vit (directory where are velocities ux, uy with and without rotation. Generally, it is the same path as path_out), derot (=1 : velocities without solar rotation; = 0 : velocities with solar rotation), nombre_j (number of days)

3. With SSWIDL, type the following commands:
.r step3_CST_IDL_apres.pro
.r step3_CST_IDL_avant.pro (depending of the treated case)

For example for 6 days HMI observations, the output files are for each day after and before :
- Vr_Vtheta_Vphi_derot_day1_ap.dat
- Vr_Vtheta_Vphi_derot_day2_ap.dat
- Vr_Vtheta_Vphi_derot_day3_ap.dat
- Vr_Vtheta_Vphi_derot_day1_av.dat
- Vr_Vtheta_Vphi_derot_day2_av.dat
- Vr_Vtheta_Vphi_derot_day3_av.dat

Warning: IDL codes of step 3 use the same "temporary" directory (/tmp) when reading SDO data. So you should not run the IDL code for 2 data sets at the same time, otherwise the SDO decompression files (in /tmp directory) will be destroyed by the 2nd IDL run.

For example, "treated_day1_ap" directory (for 30 min HMI observations) size is 19 G and "JOB_XXXX" directory size is 590 M (in "codes_CST_Fortran" directory).

For example, "treated_day1_ap" directory (for 1 day HMI observations) size is 854 G and "JOB_4174" directory size is 11 G (in "codes_CST_Fortran" directory).

As the volume of the result data is huge ("treated_day1_ap" directory), we will give only one result file of step 2 (Fortran part) corresponding to 30 min HMI observations: "output.log" in "JOB_XXXX" directory.

6 Description of input file

We describe below variables used in param_seq_29nov2018_EOS_30mn file corresponding to "29 November, 2018" HMI data. For example, we have 40 HMI files (one file each 45 seconds) for 30 minutes of data.

The following variables are common to all HMI data :

- nconv : convolution window for smoothing starting HMI images (before segmentation). nconv = 1 (default value) is the best signal-to-noise ratio

- option = vitesse : default value
- istep = 1 (fixed value) : increment step for HMI images i.e. temporal step between 2 images (all images are taken into account)
- nb_series = 1 : default value. We have only 1 sequence of images (here 30 minutes of HMI data)
- ngmax0 : maximum number of granules per image with:
  - ngmax0 = 900000 if we use standard data (hmi.Ic_45s[***])
  - ngmax0 = 960000 if we use deconvoluted data (hmi.Ic_45s_dcon[***])
- ngmax : maximum number of granules tracked during the 30-minute sequence (i.e. 40 images) with:
  - ngmax = 25000000 if we use standard data (hmi.Ic_45s[***])
  - ngmax = 29000000 if we use deconvoluted data (hmi.Ic_45s_dcon[***])
- np = 40 : number of images corresponding to 30 minutes of HMI data
- n_ech = 40 : default value. np = n_ech
- n_life = 60 : maximum granule lifetime. A granule is tracked on maximum 60 images
- bin_sp = 7 (default value) : space bin in pixel (i.e. 2.5 Megameters)
- v_thres = 7.0 (fixed value) : speed of sound in photosphere (maximum speed in km/s considered here)
- t_thres = 180. : minimum granule lifetime in seconds. 180 s correspond to 4 HMI images. If a granule is only tracked in 3 images, then it is not taken into account. t_thres = 180 is the best signal-to-noise ratio
- seuil_min = 0.01 (default value) : we consider here all pixels
- segmentation_type = strouss (default value) : we don't use Strouss segmentation method here but CST segmentation method
- suffix = .fits : file format
- verbose = 1 : computation details printed in output_XXXX.log file (in JOB_XXXX directory). If verbose = 0, there is no output.log

The folllowing variables are corresponding to 29 November, 2018 HMI data :

- prefix : path to "day1_apres_int_derot_" FITS files (results of IDL Step 1)
- input_file = param_seq_29nov2018_EOS_30mn

- arcsec = 715.403 : value of 1 arcsec in km which is obtained by the following way : 715.403 = R_S(km) / R_S(arcsec), where R_S(km) = 696000 is the solar radius in km, R_S(arcsec) = 972.877991 is the solar radius in arcsec (provided by the header of the 1st HMI image (in fits format) of sequence for each day (RSUN_OBS=INDEX.RSUN_OBS))
- timestep = 45. : time (cadence) in seconds between 2 HMI images
- pixel= 0.504008 : pixel size in arcsec provided by the header of the 1st HMI image of sequence for each day (pix=INDEX.CDELT1)

7 Formulas implemented in CST codes

We refer to the scheme of CST codes (see figure 1) and explanations in sections 3.1 and 3.3.

7.1 Step 1 : CST IDL part

The first step of CST codes (see section 3.1 for more details) is divided into 9 parts. IDL files corresponding to this step are : reduction_doppler_intensity_all_days_apres.pro and reduction_doppler_intensity_all_days_avant.pro. These 9 parts are :

- computation of latitude and longitude of the entire solar disk for each observation day
- alignment and resizement of the Doppler data. In output files, North is at the bottom and file size is 4096 x 4096 pixels
- SDO motion correction : we correct satellite motion on Doppler data. In output files, North is at the top and the file size is 4096 x 4096 pixels
- limbshift correction : we measure the limbshift; we calculate circular average and we correct the Doppler data for each pixel of the Sun. In output files, North is at the top and file sizes are 586 x 586 pixels and 4096 x 4096 pixels
- average over 30 min of Doppler data: available file V_{Dop} with solar rotation. In output files, North is at the top and file size is 586 x 586 pixels
- measure of the solar rotation from Doppler data (or use the standard solar rotation)
- derotation of Doppler data (see figure 2). In output files, North is at the top and file size is 586 x 586 pixels
- average over 30 min of Doppler data: we produce V_{Dop} maps without solar rotation. In output files, North is at the top and file size is 586 x 586 pixels
- intensity alignment : recentering and derotating in the same way as before for Doppler data,
    - ⋆ with rotation (aligned and resized, North top)
    - ⋆ derotation of the intensity (North top)

Formulas implemented in IDL codes (reduction_doppler_intensity_all_days_apres.pro and reduction_doppler_intensity_all_days_avant.pro) are proved for points 3, 4 and 7 of the list above.

7.1.1 SDO motion correction

We compute HMI Doppler corrections from satellite motions V_{Cor} which correspond to :

- $V_{xyout} = V_{Obs}$-V_{Cor} in IDL subroutine (see part "SECOND STEP Correction of the SDO motions")
- formula (A.5) in appendix A of Rincon et al. (2017).

Here is a set of variables used in formulas. They are illustrated in figure 11.

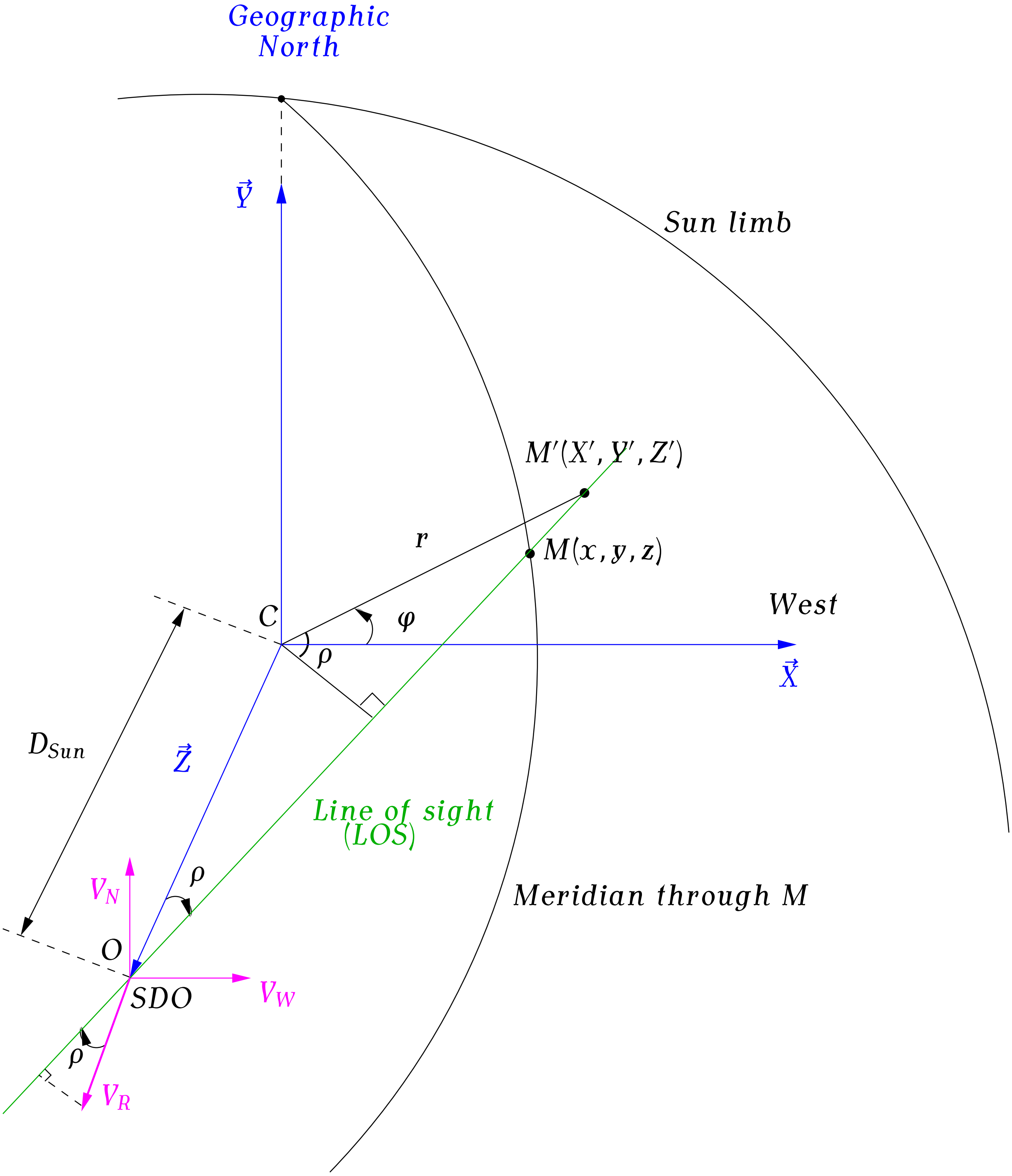

Figure 11: System of coordinates used in the computation of SDO motion correction. The observer is on the satellite. Original HMI/SDO data have the North at the bottom and East on the right of the image; the images are rotated in order to have the North on top and East on the left.

- V_W : SDO satellite relative velocity with respect to the Sun in the east-west direction, corresponds to HMI keyword “OBS_VW”, in m/s (see page 15 of document on JSOC keywords used for metadata). In CST codes, V_W is used in km/s.

- V_N : SDO satellite relative velocity with respect to the Sun in the south-north direction, corresponds to HMI keyword “OBS_VN”, in m/s (see page 15 of document on JSOC keywords used for metadata). In CST codes, V_N is used in km/s.
- V_R : SDO satellite relative velocity with respect to the Sun in radial direction, corresponds to HMI keyword “OBS_VR”, in m/s (see page 15 of document on JSOC keywords used for metadata). In CST codes, V_R is used in km/s.
- D_{Sun} : distance from the satellite to the center of the Sun in m (according to page 15 of document on JSOC keywords used for metadata). In CST codes, D_{Sun} is used in km.
- M(X,Y,Z) : a point on Solar sphere
- M’(X’,Y’,Z’) : the projection of point M along the line of sight in plan $(\vec{X}, \vec{Y})$
- C : center of the Sun’s disk
- O : origin of reference (V_W, V_N, V_R) and position of SDO satellite
- OM’ : line of sight (LOS)
- CM’ : projection of (OM’) of the LOS in North-West plan $(\vec{X}, \vec{Y})$
- φ : angle of the point M from east-west direction
- ρ : viewing angle of point M from disk center
- r : distance between point C and point M’

In North-West plan :

According to figure 12 :

- $V_N \ \cos\left(\frac{\pi}{2} - \varphi\right) = V_N \ \sin\varphi$ is the projection of V_N on the LOS
- $V_W \ \cos\varphi$ is the projection of V_W on the LOS

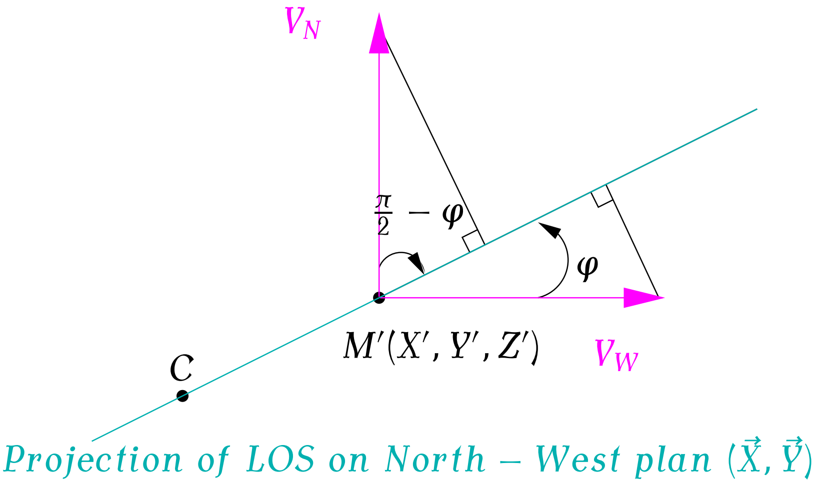

Figure 12: Projections of V_N and V_W on the projected LOS in North-West plan $(\vec{X}, \vec{Y})$.

According to figure 11, HMI Doppler correction to be applied at point M' can be written as :

$$\begin{aligned} &\cos\left(\frac{\pi}{2} - \rho\right) \times (V_W \ \cos\varphi + V_N \ \sin\varphi) \\ &= \sin\rho \times (V_W \ \cos\varphi + V_N \ \sin\varphi) \end{aligned} \tag{7.1}$$

In Radial-West plan :

According to figure 11 :

$$-V_R \ \cos\rho \ \text{ is the projection of } V_R \text{ on the LOS.} \tag{7.2}$$

By combining 7.1 and 7.2, HMI Doppler corrections from satellite motions V_{Cor} can be written as :

$$V_{Cor} = -V_R \ \cos\rho + \sin\rho \times (V_W \ \cos\varphi + V_N \ \sin\varphi) \tag{7.3}$$

In IDL subroutine, V_{Cor} is such that $V_{xyout} = V_{Obs}$-V_{Cor}.

Remark 7.1 *In the Solar reference, the observed Doppler velocity V_{Dop} is such that :*

$$V_{Dop} = V_{Obs} + V_{Cor}$$

In fact, we want the apparent motion of the Sun from the satellite. So we must take off $-V_{Cor}$ because $-V_{Cor}$ is the satellite motions relatively to the Sun. So,

$$V_{DopCor} = V_{Obs} - V_{Cor}$$

Proof of formula (A.5) in appendix A of Rincon et al. (2017)

According to figure 13,

$$\cos\varphi = \frac{X'}{r}$$
$$\sin\varphi = \frac{Y'}{r} \tag{7.4}$$

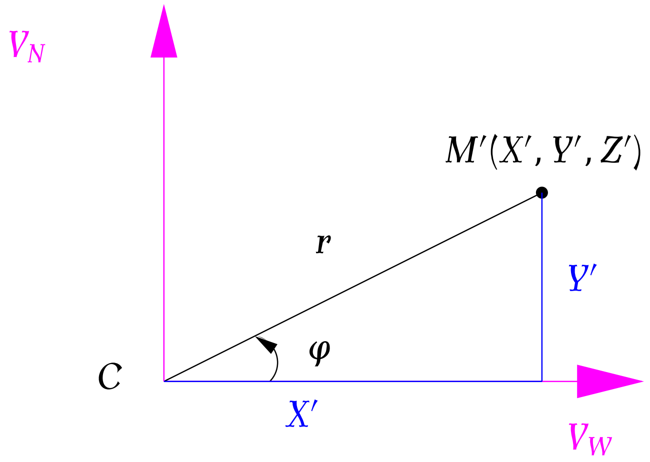

Figure 13: Scheme

According to figure 11, as OCM' is a right-angle triangle ($\widehat{OCM'} = \frac{\pi}{2}$), we have :

$$\sin\rho = \frac{r}{D_{Sun}}, \tag{7.5}$$

with $r = \sqrt{X'^2 + Y'^2}$.

So, according to (7.4) and (7.5), equation (7.3) of V_{Cor} can be written as :

$$V_{Cor} = -V_R\ \cos\rho + \frac{X'}{D_{Sun}}\ V_W + \frac{Y'}{D_{Sun}}\ V_N$$

which correponds to $-u^{sat}_{Dop}$ of formula (A.5) in appendix A of Rincon et al. (2017).

7.1.2 Correction of Doppler data amplitude related to the derotation of data

In previous section, we have corrected motions of the satellite at each point of velocity.
In this section, we compute correction of amplitude of Solar rotation on Doppler velocities, whic corresponds to :

- a part of variable $omega\_correction$ in IDL subroutine (see part "STEP 6 Derotation of the Doppler"),
- formula (A.4) in appendix A of Rincon et al. (2017).

Here is a set of variables used in formulas. They are illustrated in Figures 14 and 15.

- $(\vec{X}, \vec{Y})$: observationnal plan
- M : a point observed on the sky plan $(\vec{X}, \vec{Y})$
- Ω : differential rotation of the Sun
- $V(V_x, V_z)$: tangential velocity at point M
- $R_\odot$: solar radius. In CST codes, $R_\odot$ is used in pixels.
- lat : angle along the latitude
- lon : angle along the longitude
- B_0 : angle of inclination of the Sun or latitude of disk center

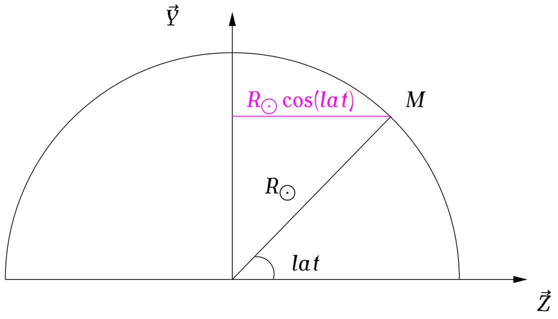

Figure 14: Section of the Sun along the central meridian, in plan $(\vec{Y}, \vec{Z})$.

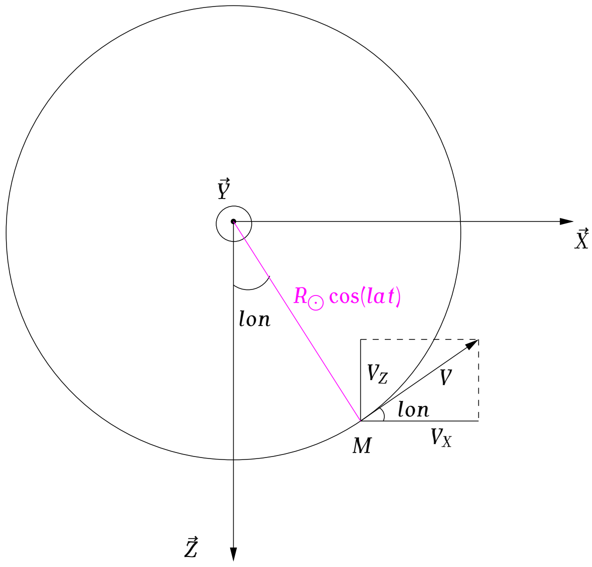

Figure 15: Sun viewed from the North at a given latitude.

According to figures 14 and 15 :

- $R_\odot \cos(lat)$ is the projection of Solar radius on plan $(\vec{Y}, \vec{Z})$ at a given latitude
- $V = \Omega(lat)\ R_\odot \cos(lat)$
- $V_X = V \cos(lon)$
- $V_Z = V \sin(lon) \cos(B_0)$

Hence, correction of amplitude of Solar rotation on Doppler velocities is written as :

$$V_Z(lat) = \Omega(lat)\ R_\odot \cos(lat) \sin(lon) \cos(B_0)$$

It corresponds to variable u_{Dop}^{rot} in formula (A.4) in appendix A of Rincon et al. (2017). And V_Z is in variable omega_correction in IDL subroutine (see part "STEP 6 Derotation of the Doppler").

7.1.3 Limbshift correction

We compute the heliocentric angle from disk center ρ which corresponds to :

- variable rho in IDL subroutines (see "STEP 3 limbshift correction"),
- formulas (A.7) in appendix A of Rincon et al. (2017).

Here is a set of variables used in formulas. They are illustrated in figures 16 and 17.

- θ : here is the colatitude. Warning : in CST codes, θ is used as latitude
- φ : longitude
- B_0 : angle of inclination of the Sun or latitude of disk center
- $(\vec{x},\ \vec{y},\ \vec{z})$: Sun reference
- $(\vec{X},\ \vec{Y},\ \vec{Z})$: observation reference
- r : distance between point M and disk center
- ρ : heliocentric angle from disk center
- $M(x, y, z)$: a point on solar disk in Sun reference $(\vec{x},\ \vec{y},\ \vec{z})$
- $R_\odot$: solar radius

According to figure 16, we have $r = \sqrt{x^2 + y^2}$. According to figure 17, we have :

- $x = X$
- $y = Y \cos B_0 - Z \sin B_0$

Spherical coordinates can be written as :

$$\begin{aligned} X &= R_{\odot} \sin\theta \sin\varphi \\ Y &= R_{\odot} \cos\theta \\ Z &= R_{\odot} \sin\theta \cos\varphi \end{aligned}$$

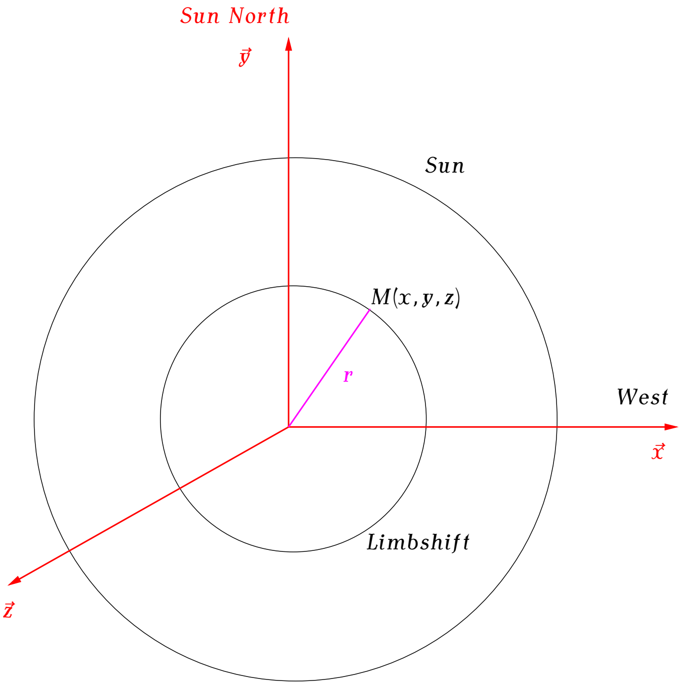

Figure 16: Sun viewed from the pole. $(\vec{x}, \vec{y}, \vec{z})$ is Sun reference. $M(x, y, z)$ is a point of the solar disk.

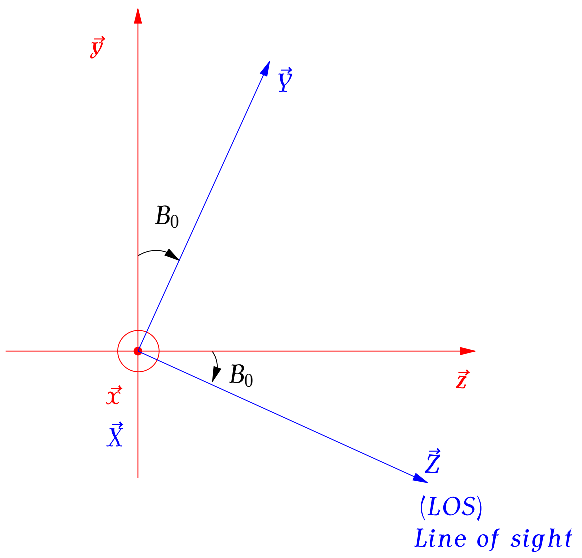

Figure 17: System of coordinates used in the computation of limbshift correction.

So, r can be written as :

$$r = R_\odot \sqrt{\sin^2\theta \sin^2\varphi + (\cos\theta\cos B_0 - \sin\theta\cos\varphi\sin B_0)^2}$$

According to figure 18, we have :

$$\sin\rho = \frac{r}{R_\odot} \Longleftrightarrow \rho = \arcsin\left(\frac{r}{R_\odot}\right)$$

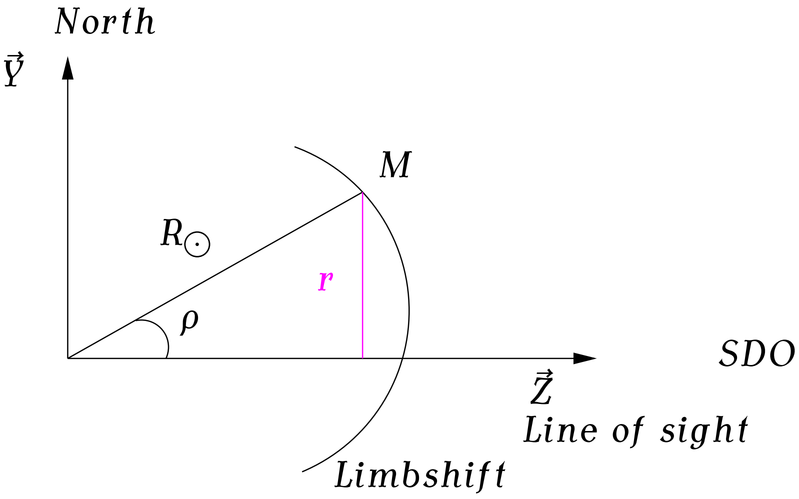

Figure 18: Heliocentric angle from disk center ρ.

Then, the heliocentric angle from disk center ρ can be written as :

$$\rho = \arcsin\sqrt{\sin^2\theta \sin^2\varphi + (\cos\theta\cos B_0 - \sin\theta\cos\varphi\sin B_0)^2}$$

It corresponds to rho in IDL subroutines (see “STEP 3 limbshift correction”) and θ is used as latitude.

Limbshift correction is given by u_{Dop}^{lim} in formula (A.6) of appendix A of Rincon et al. (2017) and is implemented in IDL subroutines (see “STEP 3 limbshift correction”) as Vdop_cor :

$$u_{Dop}^{lim} = (-0.35\,x + 0.2\,x^2 + 0.46\,x^3)$$

where $x = 1 - \cos\rho$.

7.1.4 Solar derotation applied to Doppler and intensity data

In this section, we calculate the derotation of the Sun dx, dy with respect to the reference meridian.
Here is a set ot variables used in formulas. They are illustrated in Figures 19, 20 and 21.

- B_0 : angle of inclination of the Sun
- Ω : solar rotation in rad/s depending on the latitude
- R : solar radius
- M : a point on solar disk at a given latitude λ and longitude φ
- $V(V_x, V_y)$: velocity at the solar surface for a given latitude and longitude
- V_{S_r} : velocity of solar rotation along the line of sight (LOS)

We start by calculating V_x and V_y.

Computation of V_x and V_y (Howard and Harvey, 1970) :

Case of $B_0 = 0$:

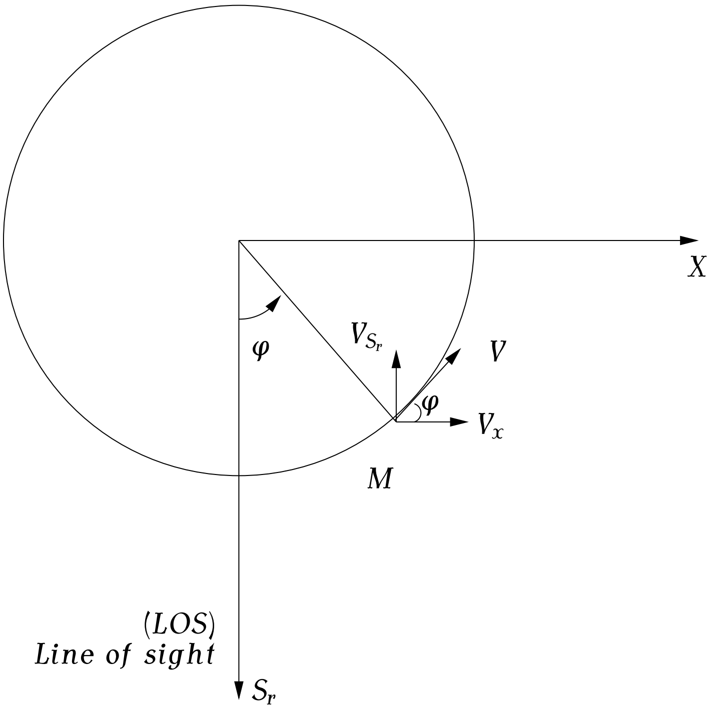

Figure 19: View from the North pole ($B_0 = 0$)

According to Figure 20, $V = \Omega\ R\ \cos\lambda$.
According to Figure 19, $V_x = V\cos\varphi$ and $V_{S_r} = V\sin\varphi$.
Therefore, $V_x = \Omega\ R\ \cos\lambda\ \cos\varphi$ and $V_{S_r} = \Omega\ R\ \cos\lambda\ \sin\varphi$.

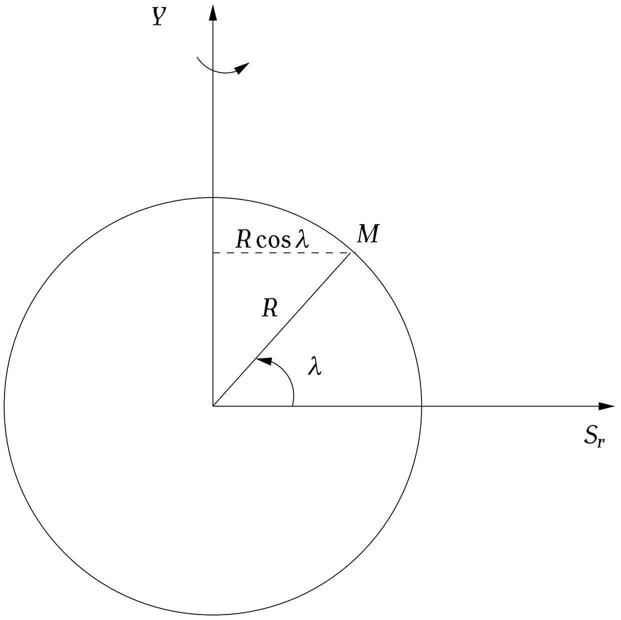

Figure 20: Side view ($B_0 = 0$)

Case of $B_0 \neq 0$:

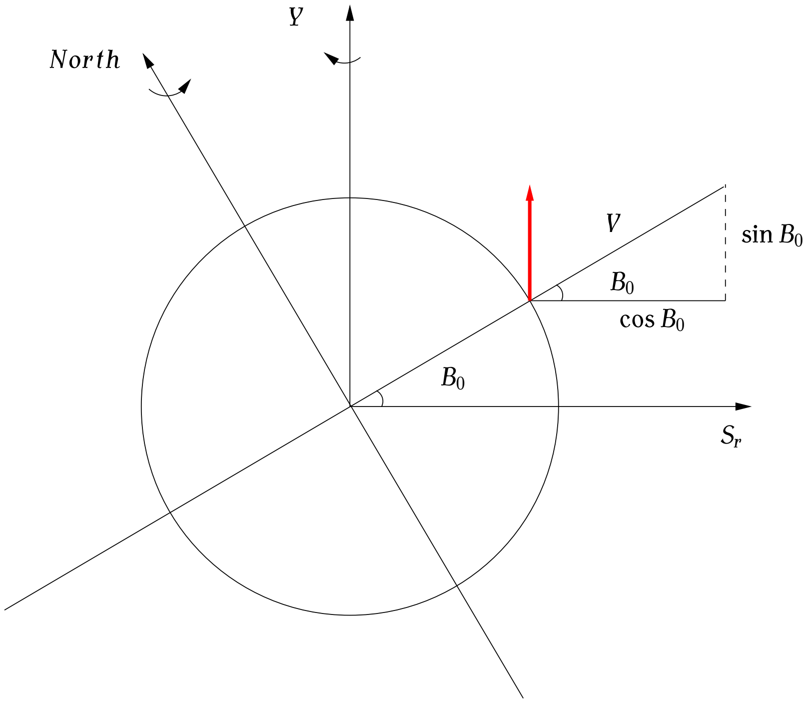

Figure 21: Case of $B_0 \neq 0$

According to Figure 21, V_x is unchanged and $V_{S_r} = \Omega\ R\ \cos\lambda\ \sin\varphi\ \cos B_0$.
We introduce a component parallel to the Y axis (see the red arrow on Figure 21).

Then, $V_y = V \sin\varphi \sin B_0$.
Finally, $V_y = \Omega\ R\ \cos\lambda\ \sin\varphi\ \sin B_0$.

Computation of the Sun derotation dx and dy with respect to the reference meridian :

Let ϕ_0 be the reference angle for which we want to bring back and $\phi(t) = \phi_0 + \Omega.t$. Let λ (resp. φ) be the latitude (resp. longitude) of a given point on solar disk.
According to Figure 22, for $B_0 \neq 0$, we can write $V_x(t) = \Omega\ R\ \cos\lambda\ \cos\phi(t) = \Omega\ dx(t)$ with

$$dx(t) = \int_{\phi_0}^{\phi_0+\Omega.t} R\ \cos\lambda\ \cos\phi(t)\ dt$$
$$dx(t) = R\ \cos\lambda \int_{\phi_0}^{\phi_0+\Omega.t} \cos\phi(t)\ dt$$
$$dx(t) = R\ \cos\lambda\ [\sin(\phi_0 + \Omega.t) - \sin\phi_0]$$

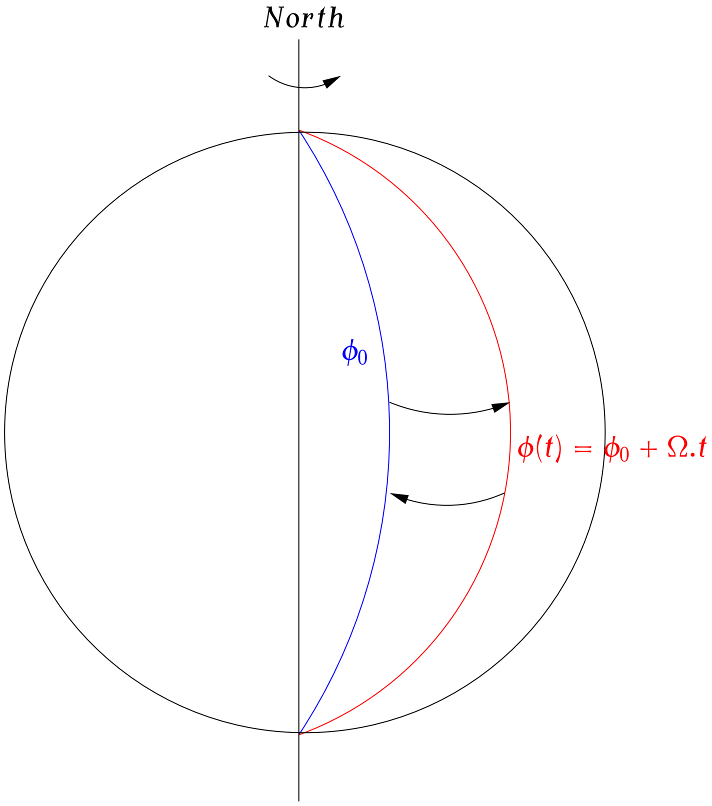

Figure 22: ϕ_0 has become ϕ. We bring back to ϕ_0 and apply dx back. Here, $B_0 \neq 0$

We have seen previously: $V_y(t) = \Omega\ R\ \cos\lambda\ \sin\varphi\ \sin B_0 = \Omega\ dy(t)$ with

$$\begin{aligned}
dy(t) &= \int_{\phi_0}^{\phi_0+\Omega.t} R\ \cos\lambda\ \sin\phi(t)\ \sin B_0\ dt \\
dy(t) &= R\ \cos\lambda\ \sin B_0 \int_{\phi_0}^{\phi_0+\Omega.t} \sin\phi(t)\ dt \\
dy(t) &= R\ \cos\lambda\ \sin B_0\ [-\cos(\phi_0+\Omega.t)+\cos\phi_0] \\
dy(t) &= -R\ \cos\lambda\ \sin B_0\ [\cos(\phi_0+\Omega.t)-\cos\phi_0]
\end{aligned}$$

In conclusion, to derotate the solar image for the CST, we used $dx(t)$ and $dy(t)$ formulae. These formulae are implemented in STEP 1 of CST IDL part:
see reduction_doppler_intensity_all_days_apres.pro file, in particular "STEP 6 : Derotation of the Doppler" (dxx and dyy variables) and "STEP 9: Derotation of the intensity" (dxx and dyy variables).

7.2 Step 3 : CST IDL part

The third step of CST codes (see section 3.3 and figure 1 for more details) is divided into 4 parts. IDL files corresponding to this step are : step3_CST_IDL_apres.pro and step3_CST_IDL_avant.pro. These 4 parts are :

- transformation of binary data $(U_x,\ U_y)$ into fits data $(V_x,\ V_y)$
- computation of SDO motion correction parameters for V_x and V_y
- correction if the SDO motions on V_x and V_y, with and without the differential rotation of the Sun
- transformation of V_x, V_y, V_{Dop} into V_r, V_θ, V_φ.

Formulae implemented in IDL codes (step3_CST_IDL_apres.pro, step3_CST_IDL_avant.pro) are proved for points 2 and 4 of the list above.

7.2.1 Computation of SDO motion correction parameters to apply to V_x and V_y

We compute SDO motion correction parameters V_{xCor} and V_{yCor} which correspond to :

- variables V_{xCor} and V_{yCor} in IDL subroutine step3_CST_IDL_apres.pro,
- formulas (A.2) and (A.3) in appendix A of Rincon et al. (2017).

Here is a set of variables used in formulas. They are illustrated in figure 23.

- V_W : SDO satellite relative velocity with respect to the Sun in the east-west direction, corresponds to HMI keyword "OBS_VW", in m/s (see page 15 of document on JSOC keywords used for metadata). In CST codes, V_W is used in km/s.
- V_N : SDO satellite relative velocity with respect to the Sun in the south-north direction, corresponds to HMI keyword "OBS_VN", in m/s (see page 15 of document on JSOC keywords used for metadata). In CST codes, V_N is used in km/s.

- Ω_W : relative angular rotation of the satellite with respect to the east-west direction;

$$\Omega_W = \frac{V_W}{D_{Sun}}$$

- Ω_N : relative angular rotation of the satellite with respect to the south-north direction;

$$\Omega_N = \frac{V_N}{D_{Sun}}$$

- D_{Sun} : distance between the satellite and the center of the Sun in m (according to page 15 of document on JSOC keywords used for metadata). In CST codes, D_{Sun} is used in km.

- $R_{\odot}$: Solar radius in km for analytic calculations. In CST codes, $R_{\odot}$ is used in pixels. Keywords used for $R_{\odot}$ to have it in pixels are :
  - pix = CDELT1 : pixel size in arcsec
  - RSUN_OBS = RSUN_OBS : Solar Radius in arcsec
  - Rsun = RSUN_OBS/pix : Solar Radius in pixel

- M(X,Y,Z) : a point on Sun surface.

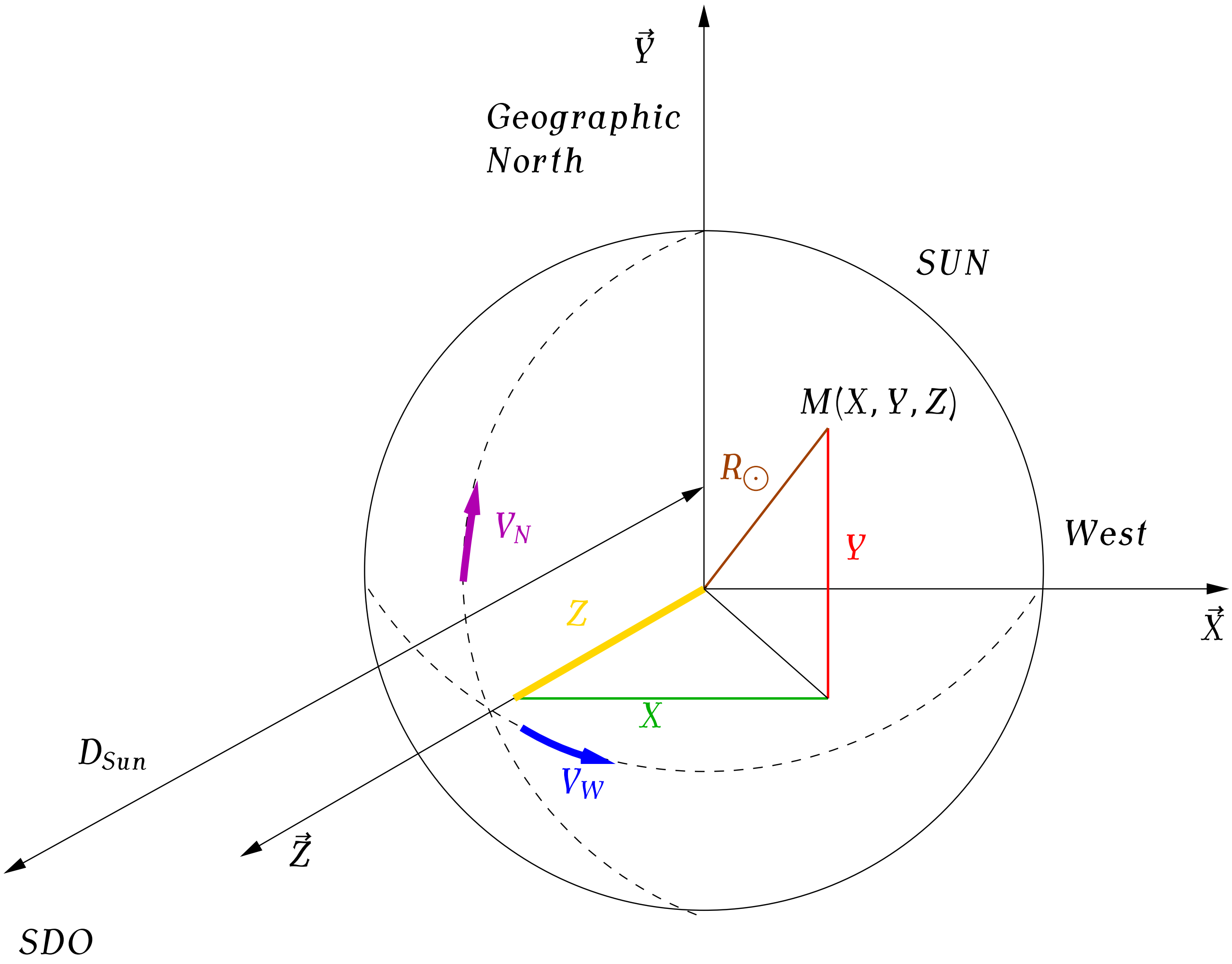

Figure 23: System of coordinates used in the computation of SDO motion correction parameters. The observer is on the satellite. Original HMI/SDO data have the North at the bottom and East on the right of the image; the images are rotated in order to have the North on top and East on the left.

Solar radius is written as: $R_\odot^2 = X^2 + Y^2 + Z^2$, so that $Z = \sqrt{R_\odot^2 - (X^2 + Y^2)}$.

According to figure 24, $R_\odot = \dfrac{Y}{\sin\alpha}$ and $Z = \dfrac{Y}{\tan\alpha}$.

Then, SDO motion correction parameter to apply to V_y can be written as :

$$\begin{aligned} V_{yCor} &= -\Omega_N\ R_\odot.\cos\alpha = -\Omega_N \frac{Y}{\sin\alpha}\cos\alpha \\ &= -\frac{\Omega_N}{\tan\alpha}\ Y = -\Omega_N\ Z = -\Omega_N\sqrt{R_\odot^2 - (X^2+Y^2)} \\ V_{yCor} &= -\frac{V_N}{D_{Sun}}\sqrt{R_\odot^2 - (X^2+Y^2)}. \end{aligned}$$

In the same way, SDO motion correction parameter to apply to V_x is written as :

$$V_{xCor} = -\Omega_W \, Z = -\frac{V_W}{D_{Sun}} \sqrt{R_\odot^2 - (X^2 + Y^2)}.$$

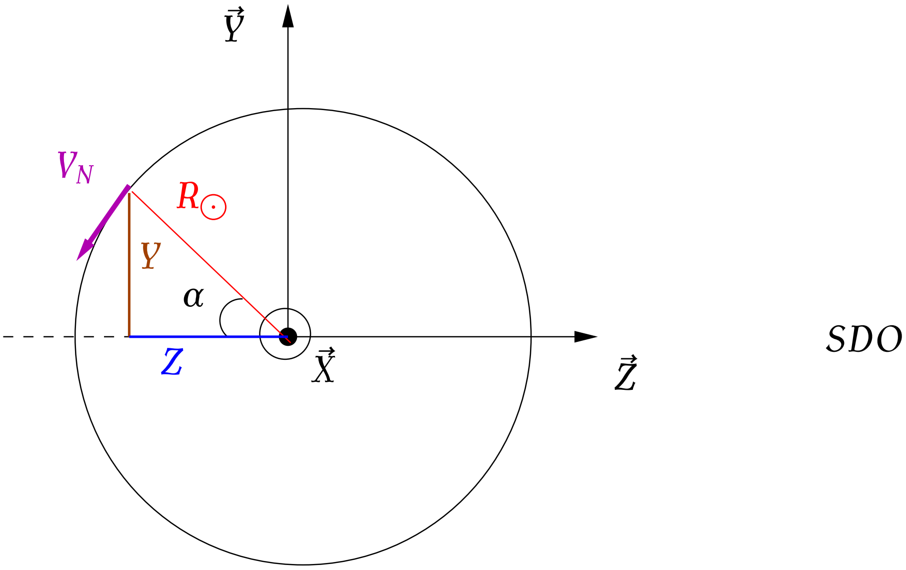

Figure 24: System of coordinates used in the computation of SDO motion correction parameters. The observer is on the satellite.

7.2.2 Transformation of V_x, V_y, V_{Dop} into V_r, V_θ, V_φ

In this section, formulas of V_r, V_θ, V_φ are proved. They are corresponding to variables v_r, v_phi and v_theta in IDL file (step3_CST_IDL_apres.pro) and formulas written in section 5 of Roudier et al. (2013).

Here is a set of variables used in formulas. They are illustrated in figure 25.

- θ : heliographic latitude
- φ : heliographic longitude
- b : angle of inclination of the Sun (B_0) or latitude of disk center
- $(\vec{x}, \vec{y}, \vec{z})$: Sun reference
- $(\vec{X}, \vec{Y}, \vec{Z})$: observation reference
- V_x, V_y, V_{Dop} : velocity components at point M in the observationnal coordinate sytem of the figure 23
- V_r, V_θ, V_φ : velocity components at point M in Sun reference.

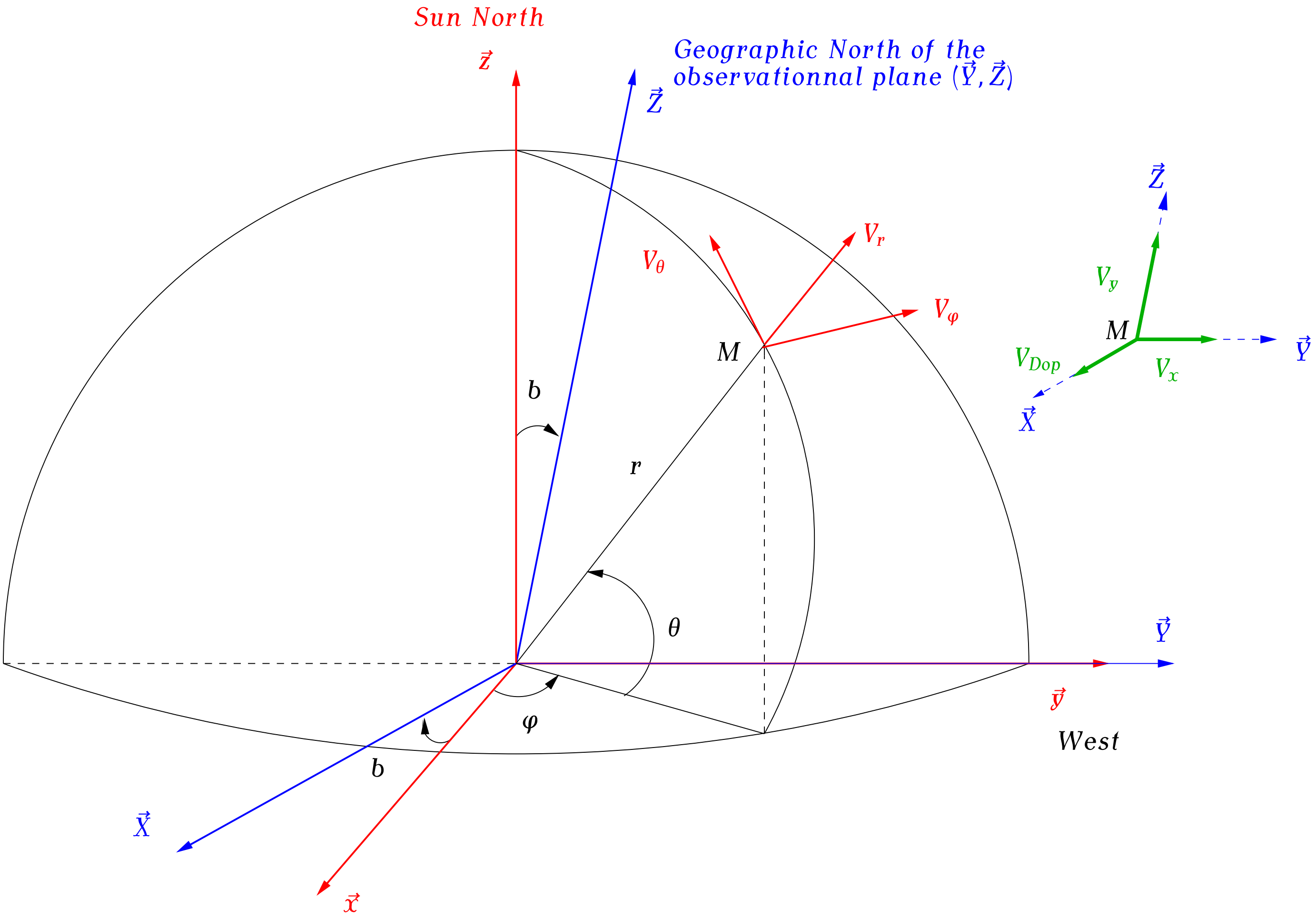

Figure 25: Coordinate system used in formulas : velocity components (V_x, V_y, V_{Dop}) at point M in observation reference ($\vec{X}$, $\vec{Y}$, $\vec{Z}$) and velocity components (V_r, V_θ, V_φ) at point M in Sun reference ($\vec{x}$, $\vec{y}$, $\vec{z}$).

To connect ($\vec{x}$, $\vec{y}$, $\vec{z}$) to observation plan ($\vec{Y}$, $\vec{Z}$), the following rotations must be applied :

- a rotation with respect to $\vec{y}$ of angle $-b$ (see matrix A)
- a rotation with respect to $\vec{z}$ of angle φ (see matrix B)
- a rotation with respect to $\vec{y}$ of angle θ (see matrix C),

with

$$A = \begin{pmatrix} \cos b & 0 & \sin b \\ 0 & 1 & 0 \\ -\sin b & 0 & \cos b \end{pmatrix}$$

$$B = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

According to figure 25 :

- V_x is observed along $\vec{Y}$
- V_y is observed along $\vec{Z}$
- V_{Dop} is observed along $\vec{X}$.

So we obtain :

$$\begin{pmatrix} V_{Dop} \\ V_x \\ V_y \end{pmatrix} = A\ B\ C \begin{pmatrix} V_r \\ V_\varphi \\ V_\theta \end{pmatrix}$$

Transformation of V_x, V_y, V_{Dop} into V_r, V_θ, V_φ can be written as :

$$\begin{pmatrix} V_r \\ V_\varphi \\ V_\theta \end{pmatrix} = (A\ B\ C)^{-1} \begin{pmatrix} V_{Dop} \\ V_x \\ V_y \end{pmatrix}$$

We calculate $(A\ B\ C)^{-1} = C^{-1}\ B^{-1}A^{-1}$, with

$$\begin{aligned} A^{-1} &= \frac{1}{det(A)}\ {}^tCo(A) \\ B^{-1} &= \frac{1}{det(B)}\ {}^tCo(B) \\ C^{-1} &= \frac{1}{det(C)}\ {}^tCo(C), \end{aligned}$$

where $Co(A)$ (respectively $Co(B)$ and $Co(C)$) is the adjugate matrix or cofactor matrix of A (respectively B and C). $det(A)$ (respectively $det(B)$ and $det(C)$) is the determinant of matrix A (respectively B and C).

We have $det(A) = det(B) = det(C) = 1$ and

$$Co(A) = \begin{pmatrix} \cos b & 0 & \sin b \\ 0 & 1 & 0 \\ -\sin b & 0 & \cos b \end{pmatrix}$$

$$Co(B) = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Co(C) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

Then,

$$A^{-1} = \begin{pmatrix} \cos b & 0 & -\sin b \\ 0 & 1 & 0 \\ \sin b & 0 & \cos b \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C^{-1} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

We obtain : $(A\ B\ C)^{-1} =$

$$\begin{pmatrix} \cos b\cos\theta\cos\varphi + \sin\theta\sin b & \cos\theta\sin\varphi & \sin\theta\cos b - \cos\theta\cos\varphi\sin b \\ -\sin\varphi\cos b & \cos\varphi & \sin b\sin\varphi \\ \cos\theta\sin b - \sin\theta\cos\varphi\cos b & -\sin\varphi\sin\theta & \sin\theta\cos\varphi\sin b + \cos\theta\cos b \end{pmatrix}$$

Transformation of V_x, V_y, V_{Dop} into V_r, V_θ, V_φ :

$$\begin{pmatrix} V_r \\ V_\varphi \\ V_\theta \end{pmatrix} = (A\ B\ C)^{-1} \begin{pmatrix} V_{Dop} \\ V_x \\ V_y \end{pmatrix}$$

can be written as :

$$\begin{aligned} V_r &= (\cos b\cos\theta\cos\varphi + \sin\theta\sin b)\ V_{Dop} + \cos\theta\sin\varphi\ V_x \\ &+ (\sin\theta\cos b - \cos\theta\cos\varphi\sin b)\ V_y \\ V_\varphi &= -\sin\varphi\cos b\ V_{Dop} + \cos\varphi\ V_x + \sin b\sin\varphi\ V_y \\ V_\theta &= (\cos\theta\sin b - \sin\theta\cos\varphi\cos b)\ V_{Dop} - \sin\varphi\sin\theta\ V_x \\ &+ (\sin\theta\cos\varphi\sin b + \cos\theta\cos b)\ V_y \end{aligned}$$

8 Example: CST codes applied to 30 minutes HMI observations

Source codes are in “CST_TEST_30min” directory. You can run CST codes for any half hour. We consider here the following case : November 29, 2018 from 8:00 to 8:30. There are 41 intensity files and 41 Doppler files (provided by JSOC), in FITS format. These data are to be put in a directory named “day1_apres” (2.7 G).

The first step prepares input data for CST Fortran program (data reduction) by first running with SSWIDL “reduction_doppler_intensity_all_days_apres.pro”
(then “reduction_doppler_intensity_all_days_avant.pro”), in “codes_CST_IDL” directory. The results (in FITS format) are in “treated_day1_ap” directory (19 G).

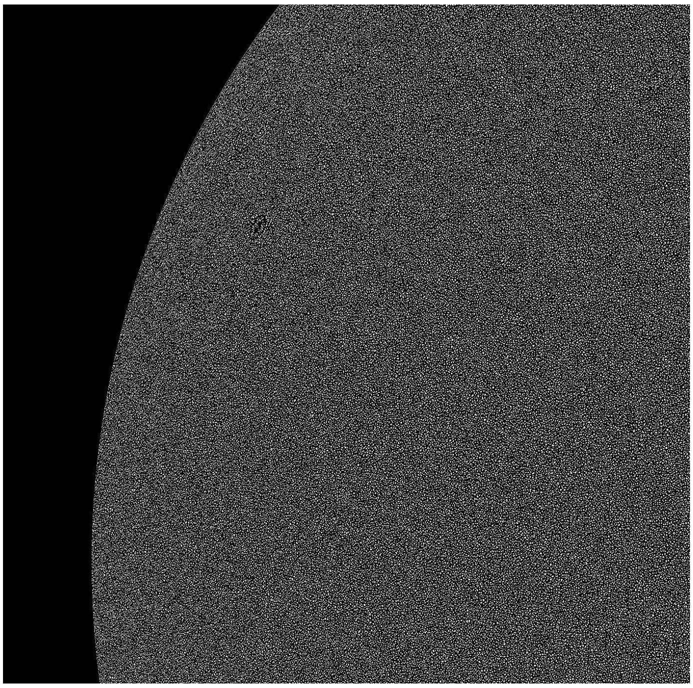

Figure 26: Solar segmented granules close to the limb - observations on October 21, 2010

You can run now the second step i.e. CST Fortran part by typing the following command, in "codes_CST_Fortran" directory, for example with SLURM :
sbatch script_29nov2018.sh

A SLURM file ("slurm-5617.out") corresponding to the JOB number (5617 in our case) is created, as well as "JOB_5617" directory which contains the results (see section 5 for more details on each result files) and "output_5617.log" file. You can check the progress of the code in "output_5617.log".
When the run of step 2 is finished, copy all ux and uy files "from JOB_5617/results" directory to "treated_day1_ap" directory.

Then, run the third step "step3_CST_IDL_apres.pro" with SSWIDL. The results are in "treated_day1_ap" directory.

figure 26 is the visualization of the binary file "image_cont".

9 Frequent troubles encountered

When you apply CST codes to 1 day HMI observations or more, sometimes it can happen that CST Fortran part (Step 2) does not calculate all the velocities ux and uy in JOB_XXX/results. Generally, one FITS file (day1_apres_int_derot_xxxx.fits) is corrupted (we don't know why) but you can overcome that problem by replacing the corrupted file by the precendent FITS file (which is good) of the series. In order to know which FITS file is corrupted (day1_apres_int_derot_xxxx.fits), you must locate it in "output_XXX.log" file. Then, you run CST Fortran program (step 2) to calculate the missing u_x and u_y. In that case, like for example NINDEX=0 , in "script_DDMMMYYY.sh", is switched into NINDEX=20 (if ux_0020 is not computed) and we also modify the line :

while [$NINDEX -le 47] to while [$NINDEX -le 20] .

10 Annex version 1.2 : solar granules deconvolution and (U_x, U_y) at higher spatial and temporal resolutions

algorithm of annex version 1.2 is divided into 3 steps :

- step 1 : IDL part which corresponds to the deconvolution of HMI/SDO intensity images (hmi.Ic_45s) with a time step of 45s (file size of each data is 4096x4096 pixels). On output, file size is 8192x8192 pixels
- step 2 : Fortran part for which mask center and radius have been multiplied by 2. On output, file size is 1172x1172 pixels (if bin_sp = 7 pixels in "param_seq_29nov2018_EOS_30mn_deconv" file, see section 10.2)
- step 3 : IDL part which corresponds to visualization of velocities and saving files (U_x, U_y)

10.1 Description of annex version 1.2

Nature of the physical problem: Measure of Solar surface velocities with higher spatial and temporal resolutions

Method of solution: Granule tracking, Daubechies wavelets

Other relevant information: HMI intensities (4092x4092 pixels) are deconvolved (8192x8192 pixels) before running Fortran part. Spatial resolution goes from 2.5 Mm to 1.75 Mm (if bin_sp = 7 pixels)

Authors: Th. Roudier, J.-M. Malherbe

Program available from: https://idoc.osups.universite-paris-saclay.fr/medoc/tools/cst-codes/

Annex version 1.2

Computer(s) on which program has been tested: IAS server

Operating System(s) for which version of program has been tested: Linux (Debian 10)

Programming language used: IDL (with **SSWIDL** software) and Fortran 90 (with **ifort** compiler)

Status: Stable

Accessibility: open (MEDOC)

Nb. of code lines in combined program and test deck: 90 and 142 lines for IDL files and 4018 lines for Fortran files (librairies not included)

Typical running time: for 15 min HMI observations and bin_sp = 7 pixels, the deconvolution takes 40 min and Fortran part takes 7 hours on IAS server using 18 cores with the following characteristics (CPU: 2 x Intel(R) Xeon(R) CPU E5-2650 v4 @ 2.20GHz 24 cores, Memory: 256Go).
For 30 min HMI observations and bin_sp = 3 pixels, IDL parts take 80 min and Fortran part takes 14 hours on IAS server using 18 cores with same characteristics than above.

10.2 Running annex version 1.2

- Download the package source file annex_version1.2.tgz from MEDOC webpage
- ifort compiler and SSWIDL software are required. Type the following linux commands:
- tar -xvzf annex_version1.2.tgz
- cd annex_version1.2
- The directory contains 2 directories corresponding to 2 different test cases to be able to check if your results are good : a first test case which corresponds to 29 Nov 2018 HMI observations of 15 min ("TEST_15min" directory), a second test case which corresponds to 29 Nov 2018 HMI observations of 30 min ("TEST_30min" directory)
- Each of these directories (corresponding to the 2 different cases above) contains the following files:

  1. Fortran files ("Fortran" directory):
     - cst_labv7_FS_2017_deconv.f90 (main program)
     - Makefile.common (used to compile librairies that are in the Lib directory)
     - deriw2d.f90
     - detect.f90
     - div_curl.f90
     - mres2d.f90
     - opt_scale.f90
     - the_name.f90 (contains thename module)
     - segment.f90 (contains segmentation module)
     - cg.f90
     - interpol2d.f90
     - interpol2dth.f90
     - interpol2d_routine.f90 (contains interpolationroutine module)
     - interpol2d_routine_short.f90
     - centre_gravite.f90

- input.f90 (contains input subroutine)
- sub_coef_r.f90
- fftw3.inc, form_num.inc, Makefile.inc
- "lib" directory contains librairies needed for compilation
- Makefile (file to modify): variables CFLAGS and CFLAGS_LIB are to be adapted according to your server (-I/usr/include/x86_64-linux-gnu/c++/8)
- IAS_cluster-head_29nov2018_script_deconv.sh (file to modify): according to your server, you need to add or not the following commands (see section 6 for more details) :
"source /opt/intel/compilers_and_libraries_2018.3.222/..." : path to Fortran ifort compiler
"ulimit -s unlimited"
- param_seq_29nov2018_EOS_30mn_deconv (file to modify) :
the following parameters are to be adapted according to the HMI/SDO data: prefix, input_file, arcsec, pixel (see section 6 for more details)

2. IDL files (IDL directory): examples with 15 min / bin_sp = 7 pixels and 30 min / bin_sp = 3 pixels
- ftm.sav : HMI transfer function
- deconvol_HMI_29nov2018.pro corresponding to deconvolution from images 4096x4096 pixels to 8192x8192 pixels
- pv_field_SDO_7_pix_15min.pro or pv_field_SDO_3_pix_30min.pro according to the test cases, corresponding to visualization of velocities (U_x, U_y)
- fits and dat files are results prooduced by pv_field_SDO_7_pix_15min.pro or pv_field_SDO_3_pix_30min.pro

- take HMI intensity and Doppler data from JSOC (45s-cadence HMI files are not available at MEDOC). For more details, see section 4. For running test case which corresponds to 29 Nov 2018 HMI observations of 15 min (resp. 30 min), we need 20 (resp. 40) intensity files.

- you must create "result_idl" directory which will contain fits files from deconvolution (STEP 1 : IDL part)

- **STEP 1: IDL PART**

  Adapt the following parameter in IDL "deconvol_HMI_29nov2018.pro" file : path, path_out. To prepare input data for Fortran program, type the following commands with SSWIDL in "IDL" directory:

  ```
  .r deconvol_HMI_29nov2018.pro
  ```

  For 15 min HMI observations, we have (in result_idl directory):

SDO_20181129_deconv_0001.fits, ..., SDO_20181129_deconv_0020.fits

For 30 min HMI observations, we have (in result_idl directory):

SDO_20181129_deconv_0001.fits, ..., SDO_20181129_deconv_0040.fits

- **STEP 2: FORTRAN PART**

  Choose 18 cores for an optimal run. To compile and execute Fortran files, here are the commands for SLURM, in "Fortran" directory:

  ```
  sbatch IAS_cluster-head_29nov2018_script_deconv.sh
  ```

  The output files are (in "JOB_XXXX/results" directory) :
  - output.log: to check if annex code runs well
  - image_cont: last segmented image of the Sun in binary format (visualized by an IDL program "image_cont.pro"(with SSWIDL : .r image_cont.pro))
  - param_seq_ddmmyyyy_EOS_30mn_deconv: parameters used
  - traject_11_0000: trajectories of all selected granules. he second number of the first line is the total number of treated granules. Column 1 is the granule number, column 2 is x_cent (gravity center), column 3 is y_cent, column 4 is the number of the image where the granule is born, column 5 is the number of the image where the granule dies, column 6 is the lifetime of the granule in second, column 7 is the velocity ux (in km/s), column 8 is the velocity uy (in km/s)
  - nb_gran_0000: pixel size chosen for the spatial window, 1 arcsec in km, pixel size in arcsec, treatment threshold of CST code
  - ux_b_0000, ux_h_0000, ux_l_0000, ux_m_0000, ux_k_0000
  - uy_b_0000, uy_h_0000, uy_l_0000, uy_m_0000, uy_k_0000
  - div_b_0000, div_h_0000, div_l_0000, div_m_0000, div_k_0000
  - rot_b_0000, rot_h_0000, rot_l_0000, rot_m_0000, rot_k_0000
  - err_b_0000, err_h_0000, err_l_0000, err_m_0000, err_k_0000
  - ux,uy,div,rot, sampled on a regular grid , traj contains the trajectories of each granule.

- **STEP 3: IDL PART**

  1. Adapt the following parameters in IDL file (in "IDL" directory):

     - pv_field_SDO_7_pix_15min.pro or pv_field_SDO_3_pix_30min.pro according to the test cases: path (path to "JOB_XXXX/results" directory)
  2. With SSWIDL, type the following commands:
     .r pv_field_SDO_7_pix_15min.pro or .r pv_field_SDO_3_pix_30min.pro according to the test cases

For example, for 15 min HMI observations and bin_sp = 7 pixels, “result_idl” directory size is 5,5 G and “JOB_XXXX” directory size is 1,4G (in “Fortran” directory).

For example, for 30 min HMI observations and bin_sp = 3 pixels, “result_idl” directory size is 11 G and “JOB_XXXX” directory size is 3,4 G (in “Fortran” directory).

10.3 Example : annex codes applied to 15 minutes HMI observations and bin_sp = 7 pixels

Source codes are in “TEST_15min” directory. We consider here the following case : November 29, 2018. In “param_seq_29nov2018_EOS_30mn_deconv” file, we consider : ngmax0 = 2000000, ngmax = 80000000, np=n_ech=20, bin_sp=7. Visualization of velocities (step 3 : IDL part) gives the following picture 27.

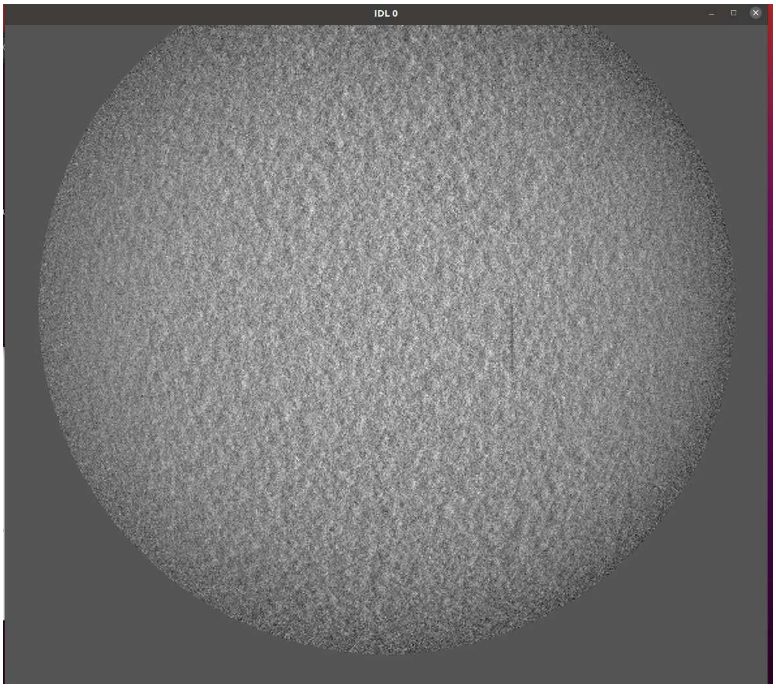

Figure 27: Solar granules close to the limb - observations on November 29, 2018

10.4 Example : annex codes applied to 30 minutes HMI observations and bin_sp = 3 pixels

Source codes are in “TEST_30min” directory. We consider here the following case : November 29, 2018. In “param_seq_29nov2018_EOS_30mn_deconv” file, we consider : ngmax0 = 2000000, ngmax = 80000000, np=n_ech=40, bin_sp=3. Visualization of velocities (step 3 : IDL part) gives the following picture 28.

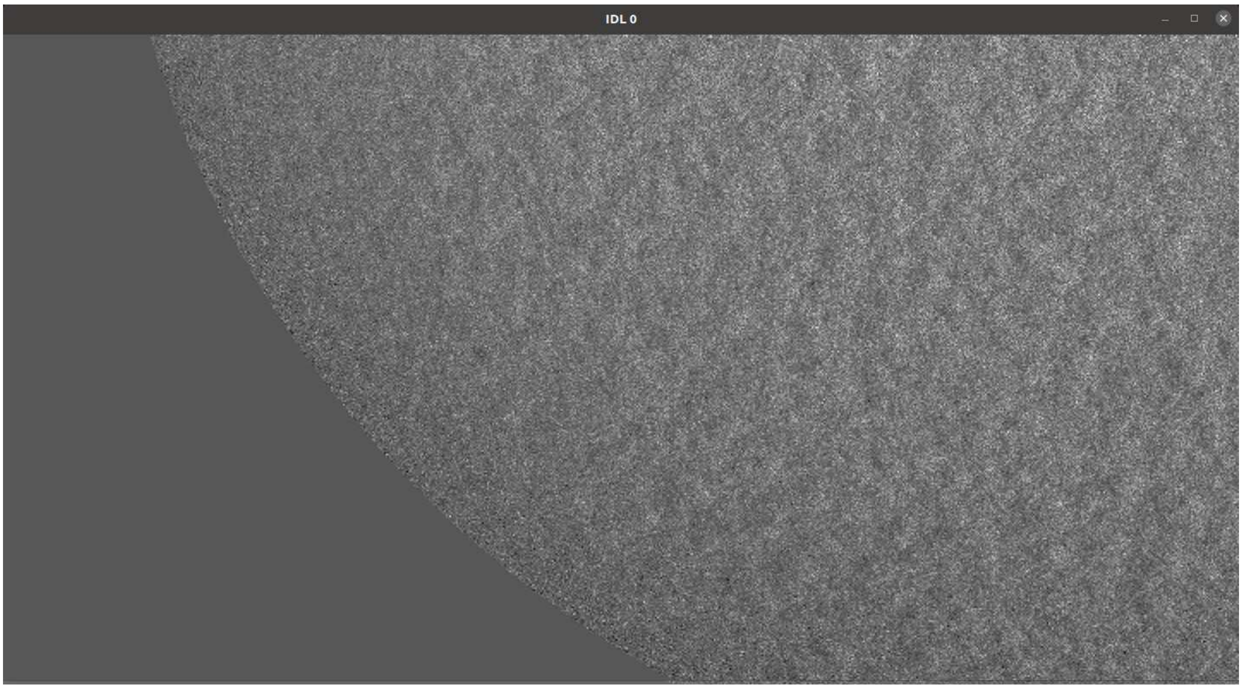

Figure 28: Solar granules close to the limb - observations on November 29, 2018

10.5 Conclusion

In section 10, we have improved the measurement of U_x and U_y by deconvoluting the original images (intensity data). This allows a gain by a factor of 2 either in time windows or in space window. The gain of going from 18 cores to 36 cores remains to be evaluated with regard to noise measurement on velocities.

We can proceed soon the same with Dopplers in order to be able to pass the whole chain of the CST for deconvolved Intensity and Doppler data. So we will obtain V_r, V_θ and V_ϕ at better spatial or temporal resolution.

11 Important note about the usage of CST codes

We strongly encourage interest scientists to use our codes. However, we shall ask them to comply with the following rules:

1. the first refereed-journal publication of any new user should include both the names of Th. Roudier (with affiliation: UniversitÃ© Paul Sabatier, Observatoire Midi-PyrÃ©nÃ©es, Cnrs, Cnes, IRAP, F-31400 Toulouse) and M. Chane-Yook (with affiliation: IAS, UniversitÃ© Paris-Saclay, Cnrs, F-91400 Orsay) among the co-authors;
2. the MEDOC service at IAS (Orsay, France) which distributes these ressources should also be properly aknowledged;
3. further publications should be mentionned to thierry.roudier@irap.omp.eu

12 Other applications of CST codes

- Measurement of cell displacement (biology)
- Measurement of any small structure displacement on a surface
- ...

13 CST MEDOC WORKSHOP

A workshop was dedicated to the training and use of the Coherent Structure Tracking (CST) software. It took place on March 3-4, 2020 at IAS in Orsay, France. This workshop is open to all researchers and students of the international solar community.

Part of the workshop was devoted to the explanation of the installation of this CST code on their own servers as well as the various precautions to take.

Workshop website: https://cst2020-medoc.sciencesconf.org/

Presentations are available on CST webpage in Workshop section.

14 Acknowledgements

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